Steam-flood modelling

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for the degree of Ph. D
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SUMMARY

The thesis describes a two-dimensional simulation model for three-phase flow, of oil, water and gas, and energy flow resulting from the injection of steam into a reservoir.

The Implicit Pressure Explicit Saturation (IMPES) method was used to solve the multiphase fluid flow, where the energy equation been solved implicitly.

The utility of the model is demonstrated for both isothermal and thermal conditions. The model predictions indicate that the explicit procedure of obtaining phase saturations strongly effects the model stability.

ACKNOWLEDGEMENT

I would like to express my sincere gratitude to Dr. M. Greaves for his assistance and encouragement during the course of this project, member of staff and colleagues at the School of Chemical Enineering, Kuwait Oil Co. for financial assistance, and last not least my family for there nonflagging support throughout these years.

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NOMENCLATURE

A	cross-sectional area of a block, ft^2
a,b,c,f,e	coefficients of the general equation
B	formation volume factor, STB/bbl
c	compressibility, psi^{-1}
c	specific heat, Btu/lb- f
D	depth, ft
erfc	complementary error function
g	acceleration due to gravity, ft/sec^2
h	thickness, ft
H	enthalpy, Btu
k	absolute permeability, darcy
Kh	thermal conductivity, Btu/hr-ft-F
Kr	relative permeability
L	distance, ft
Lv	latent heat of vaporization, Btu/lb
m	mass, lb
m	number of active grids
M	volumetric heat capacity, Btu/ft^3-F
N	number of unknowns in finite-difference scheme
NX	number of grid in x-direction
NY	number of grid in y-direction

P	pressure, psią (U, u also represent pressure)
Pbp	bubble point pressure, psia
Pwf	well flowing bottomhole pressure, psia
Pc	capillary pressure, psia
pcog	oil-gas capillary pressure, psia
pcow	oil-water capillary pressure, psia
q	flow rate, B/D
qht	heat flow or injection rate, Btu/D
r	radius, ft
re	external radius, ft
rw	well radius, ft
R	universal gas conxtant (10.723 psi-ft^3 /lb mole-R
Rs	solution gas-oil ratio, scf/STB
Rsw	gas solubility in water, scf/STB
s	skin effect
S	saturation
Sgi	initial gas saturation
Soi	initial oil saturation
Swi	initial water saturation
Sgr	residual gas saturation
Sor	residual oil saturation
Swr	irreducible water saturation
Sorst	steamflood residual Oil Saturation

t	time, day
t_D	dimensionless time
T	temperature, F (U, u also represent temperature)
T_r	reservoir temperature, F
T_{sc}	temperature, standard conditions, F
T	finite-difference transmissibility
V	volume, ft^3
V_b	bulk volume, ft^3
V_p	pore volume, ft^3
x	distance in x-direction
Δt	time increment
β	coefficient of cubic expansion, vol/vol-F
λ	mobility, darcy/cp
Λ	total mobility, darcy/cp
μ	viscosity, cp
ν	velocity, ft/day
π	constant, 3.14159
ρ	density, lb/ ft^3
Φ	potential, psia
ϕ	porosity
ω	relaxation factor
η	normal vector to the boundary

τ an acceleration parameter calculated by Equation (B-11)

ξ the normalizing parameter calculated by Equation (B-13)

OPERATORS

A coefficient matrix of a system of algebraic equation

d right-hand side vector

L lower triangular of matrix for *LU* factorisation

T transmissibility matrix

U upper tridiagonal matrix for *LU* factorisation

Δ difference operator

Δ^2 difference operation for space derivative

Δ_t difference operator for time derivative

ΔP_{LIM} maximum pressure change desired

ΔP_{MAX} maximum pressure change calculated

ΔS_{LIM} maximum saturation change desired

ΔS_{MAX} maximum saturation change calculated

SUBSCRIPTS

E east grid

f fluid

g	gas
ht	heat
i	initial
$\mp\frac{1}{2}$	boundaries of a block
M	middle grid
N	north grid
o	oil
ob	overburden
r	reservoir condition
S	south grid
s	sand
st	steam
STB	stock tank conditions
ub	underburden
w	water
w	well
W	west grid
x,y,z	direction in the Cartesian co-ordinate system

SUPERSCRIPTS

k	iteration level
n	time level, n=1,2,3....
'	derivative term (d/dp)

ABBREVIATIONS

API	Oil gravity
FDM	finit difference method
GOR	gas-oil ratio
HB	heat balance
HBE	heat balance error
IMPES	implicit pressure-explicit saturation
OIP	oil in place
LSOR	line SOR
MB	material balance
MBE	material balance error
PDE	partial differential equation
SEQ	sequential solution
SOR	successive over-relaxation
SS	simultaneous solution
WOR	Water-oil ratio
1-D	one dimensional
2-D	two dimensional
3-D	three dimensional

CHAPTER ONE

INTRODUCTION

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CHAPTER ONE
INTRODUCTION

The idea of injecting steam or hot water, as a heat carrier, in a petroleum reservoir was introduced as far back as 1931[1]. However, modern interest in steam injection did not begin until early 1960, when the Shell oil company had considerable success with cyclic injection of steam in California[2].

1.1 Steam injection processes:

Steam injection can be divided into two basic processes[3]

1. Steamflooding or steam drive
2. Cyclic steam stimulation or steam soak

1.1.1 Steam flooding

Steamflooding is an important process to improve recovery from heavy oil fields. The objective of the process is too drive the reservoir oil from the injection well toward surrounding producer wells. The path through the reservoir from injection to production wells can be visualized as four sections or zones[4, 5].

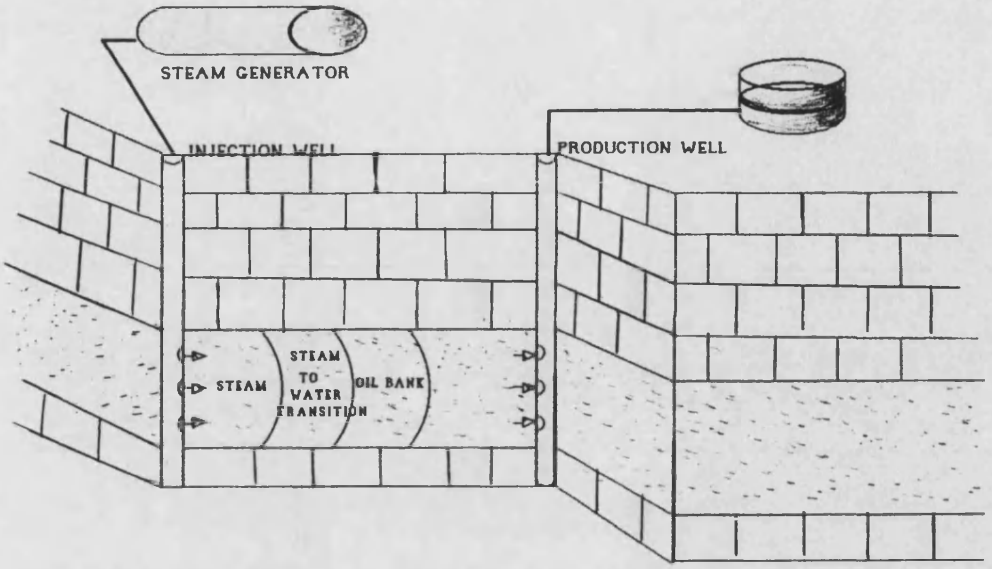

FIGURE 1.1 SCHEMATIC DIAGRAM OF STEAMFLOOD OPERATION

Steam zone:

The steam zone extends from the injection well to the leading edge of condensation zone. The temperature is reasonable uniform through this zone and equal to Ts, the steam zone temperature. The displacement of oil by steam in this region causes a great reduction in oil saturation and at certain level of steam injection , the oil saturation may reach the "Steamflood residual Oil Saturation" Sorst.[6]. Other effective mechanisms which contribute to the change in oil saturation in this zone are , thermal expansion of the oil, vaporization of volatile the hydrocarbon component and relative permeability changes.

Condensation zone:

This zone comprises the hot condensed water which creates water flood effect. The temperature of this region decreases slowly but continuously from Ts to the lower temperature of hot water zone.

Hot Water zone:

The phenomena occurring in this zone are analogous to hot waterflooding , in regard to oil displacement by hot water front. However, gravity segregation could cause steam and condensed hot water to flow at the top of this zone [7, 8].

Reservoir condition zone:

This zone is not affected by the injected heat and essentially contains the original fluid or reservoir conditions.

1.1.2 Cyclic Steam Stimulation Process:

Cyclic steam stimulation is the most widely used of the two processes of steam injection for the heavy-oil recovery, because of the fast return with relatively low initial investment [9].

Cyclic steam stimulation, or 'Huff and Puff' process consists of three distinct stages, as shown in Figure (1.2).

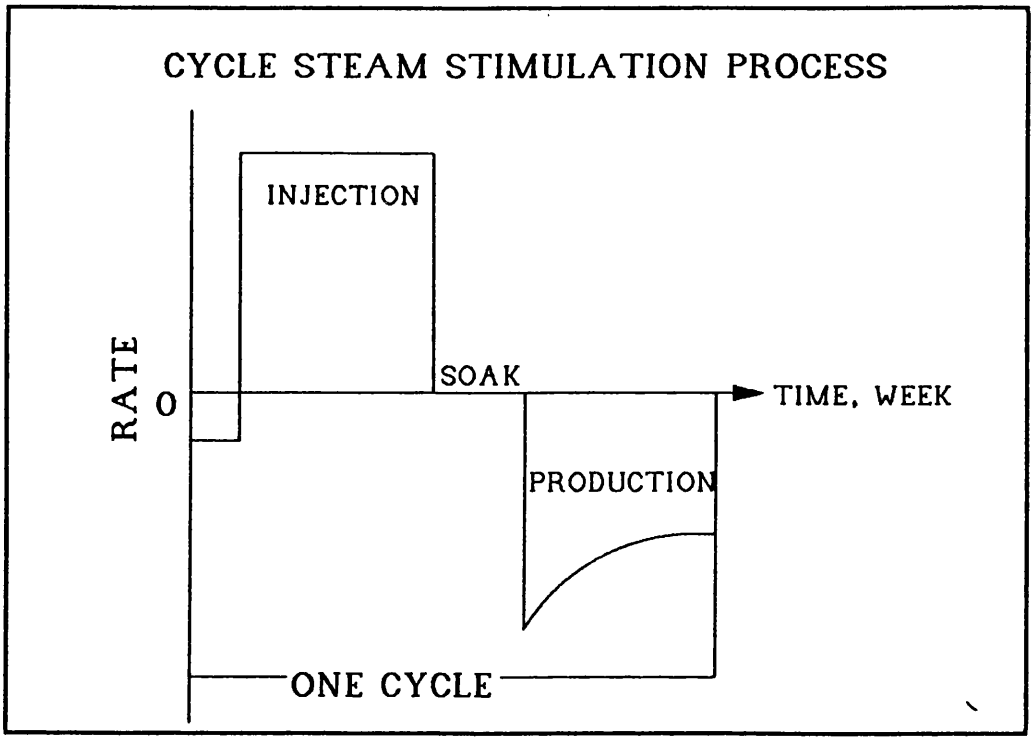

Figure 1.2 CYCLE STIMULATION PROCESS

Injection phase

During the injection phase , steam is injected at as high a rate possible into the reservoir for several weeks. Therefore the rock and the fluids around the injection wells are therefore heated significantly above original reservoir condition.

Soaking phase

In this phase, called the "soak" period, the well is shut-in for about 2 weeks. During this period, the heat penetrates further into the reservoir, transferring heat to the reservoir rock and fluids.

production phase

At this stage the well is placed on production. The oil production rate is usually considerably higher than before the injection of steam, mainly due to the oil viscosity reduction. Thermal expansion of the oil and water phases also play a significant part. Oil production is continued until it rate reaches a minimum economic rate.

The cycle (injection, soaking , and production) is repeated again as long as it remains profitable. Up to 22 cycles have been reported in Reference (9).

1.2 The Role of Steam Injection in Meeting the World Energy Demand

Steam injection method is most beneficial for recovering heavy oil and tar sand. One of the most important characteristics of heavy oil is the dramatic reduction of viscosity with a only moderate increase in reservoir temperature[10]. The effect is exponential, as represented by the Beggs and Robinson correlation[11].

$\mu_{oD} = 10^{X} - 1.0$

where:

μ = viscosity of gas-free crude (Dead oil), cp

$X = Y T^{-1.163}$

$Y = 10^{Z}$

$Z = 3.0324 - 0.02023\,API$

T = Temperature, F super degree

API = Oil gravity

Figure 1.3 shows Viscosity-Temperature curves for different types of API gravity, calculated by Beggs and Robinson correlation.

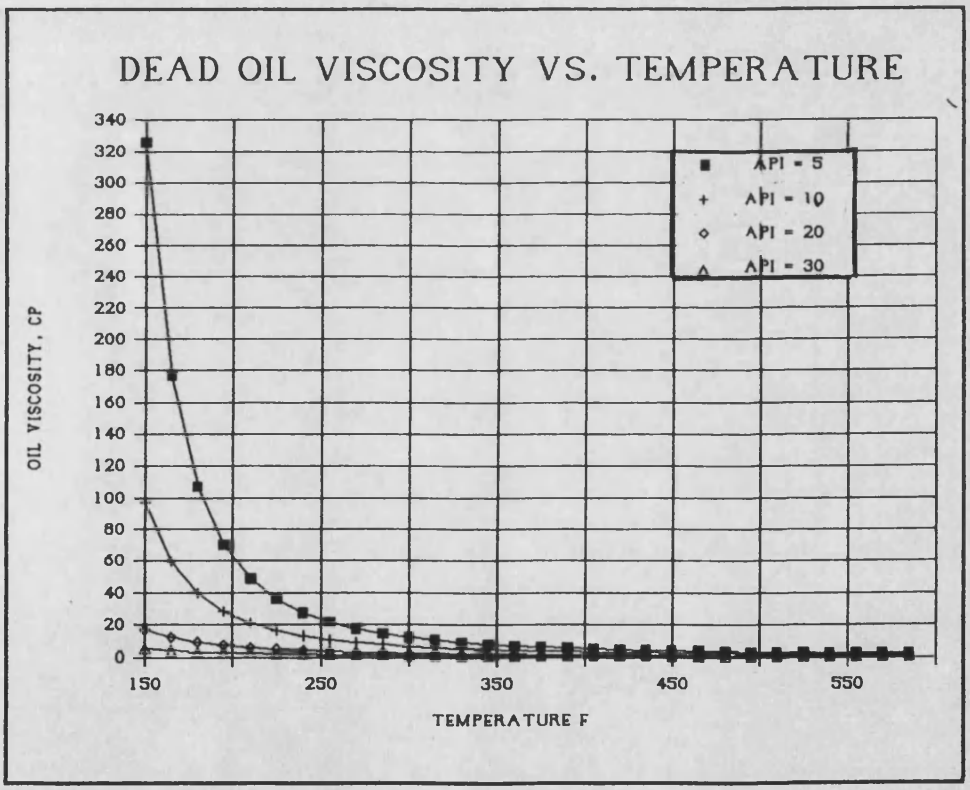

FIGURE 1.3 OIL VISCOSITY AS FUNCTION OF TEMPERATURE AND API GRAVITY

The classification of crude oil based upon API gravity is shown in Figure 1.4. Accordingly Heavy oil is defined

as having an API gravity of less than 20 API[12]. Heavy oil is also defined as having viscosity greater than 100 cp[100 mPa.s] at reservoir condition[13].

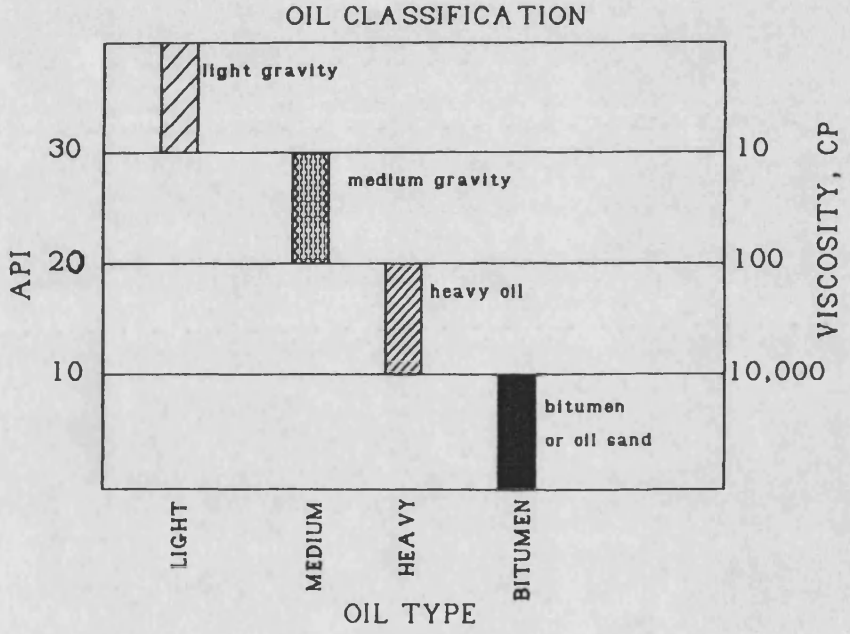

FIGURE 1.4 OIL CLASSIFICATION

The world potential of heavy oil and tar resources is very great. Ref.(12) estimated the total discovered heavy oil in place in the world to be 4,600x10^9 bbl. [730x10^9 m^3]. Table 1 shows the main knwon heavy-oil deposits in the world. Canada, Venezuela, and the Soviet Union each have of the order of one trillion bbl. of heavy oil and represent over 90% of the known heavy oil in place in the world.[14]

TABLE 1.1-MAIN KNOWN HEAVY-OIL RESOURCES	
Country	*OIP (10^9 bbl.)*
Canada	1,860
Venezuela	1,200
USSR	1,200
USA	55
Iraq	34
Iran	20
Syria	14
China	10
Ecuador	7
Trinidad and Tobago	5
Colombia	3
Others	192.0
Total	4600

TABLE 1.2-1985 PRODUCTION BY THERMAL TECHNIQUES	
Country	*Production Rate (10^3 B/D)*
USA	364
Canada	60
Venezuela	166
USSR	120
Trinidad and Tobago	70
Indonesia	40
Others	103
Total	923

Where the estimate of remaining proved probable conventional oil reserves as of Jan. 1, 1986, according to the Ref.(11), of some $700x10^9$ bbl. [$110x10^9$ m^3].
With this limited availability of light-medium crude oil, there is growing recognition that the production of heavy oil could play a major role in extending the petroleum era, which the oil industry has established over several generations, will over the next century. Exploitation of Heavy-oil could therefore guarantee the supply of fuel for the world's future energy needs, and it help also to stabilize the world political and economical market for petroleum . Table 2 shows the distribution of the 1985 world production by thermal techniques[12].

Heavy-oil recovery by steam injection requires substantial financial investment and long lead time, For productive development. Research to increase recovery efficiency is necessary to develop the full potential of the process. Economical numerical models to analyse and to predict the performance of steam injection is very important tool to aid in this process.

1.3 Simulation Models:

a simulation model is considered to be the most efficient method for predicting the performance of steam

injection processes.

In general,the objective of a simulation model is to predict future performance of a reservoir under a variety of conditions, so that an optimum set of conditions can be selected that may lead to most efficient recovery. More specifically, a simulator for steam injection process could determine the following[15]:

1. The performance of a reservoir under different rates of steam or hot water injection.
2. The effect of well location, spacing and pattern.
3. How to maximise the economic recovery under different injection and production strategies.
4. Estimate the sensitivity of the project to various input data.

1.3.1 Methodology of Simulator Development

In the development of a numerical simulator model the following steps are employed[16,17]:

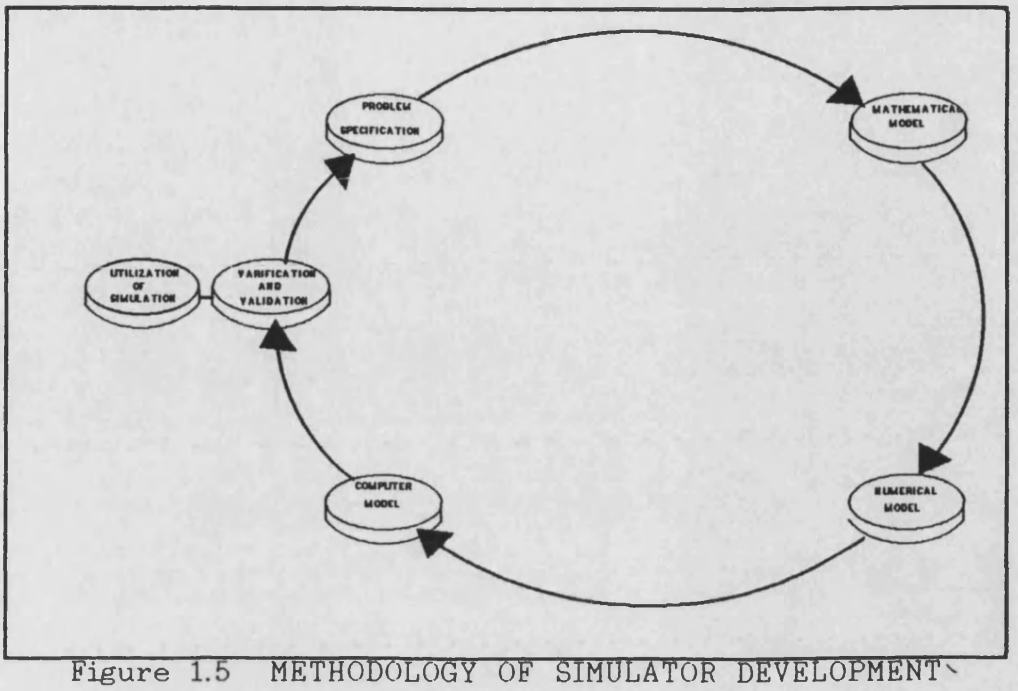

Figure 1.5 METHODOLOGY OF SIMULATOR DEVELOPMENT

1.3.1.1 Problem specification and limitations:

Accurate definition of the reservoir problem and limitations, for example, reservoir geometry, number of phases, mass and heat transfer, and input data availability, are a necessary requirements.

1.3.1.2 Mathematical modelling

In this step, the physical problem is expressed in terms of the appropriate mathematical system of equations with relevant boundary and initial conditions. The type of mathematical model depends on the following classifications:

1. Fluid Classification
   a. single phase
   b. black-oil type
   c. compositional model

2. Type of flow in porous media:
   a. Darcy type
   b. modified Darcy's law to account for high velocity effects
   c. fractured porous media model

3. Mass and heat transfer:
   a. immiscible or miscible flow model
   b. isothermal or thermal models

4. Type of geometry:
   a. one-dimensional model
   b. two-dimensional
   c. three-dimensional
   d. combination of the above models

5. Co-ordinate system:
   a. cartesian system
   b. cylindrical
   c. spherical
   d. general curvilinear
   c. combination of the above systems

6. Other capabilities
   a. well bore model
   b. surface pipeline network and equipment

The mathematical model is considered in chapter 3, using a set of nonlinear partial differential equations and appropriate boundary and initial conditions.

1.3.1.3 Numerical model

To make the problem more amenable to solution by digital computer, the mathematical model must first be transformed so as to required only arithmetic operations of addition, subtraction, multiplication, and division in its solution. In chapter 4 this approach is discussed, where the partial differential equations are transformed to numerical model using a finite difference representation. The effectiveness of the numerical method is greatly enhanced by appropriate choice of:

1. Space discretistion (5 point or 9 point)
2. The solution method for multiphase flow ,IMPES(Implicit Pressure Explicit Saturation), SEQ (SEQuential) or SS(Simultaneous Solution).
3. Time approximation.

4. Direct or iterative methods.
5. Well representation in the model.
6. Run-time controls.

1.3.1.4 Computer Model

Here, emphasis is placed on the mechanics of expressing the numerical model into a computer programme or a set of programmes. Chapter 5 describes this development.

1.3.1.5 Verification and validation

The resulting computer model is tested to determine if the model adequately simulates the reservoir problem. The model results are discussed in chapter 6.

1.3.1.6 Interpretation and utilization of simulator

After successful validation, the simulator can be utilized to study the effect of different input parameters in the original problem.

The following chapters describe the development of the numerical simulation techniques to solve the fluid flow and energy equations describing the steam injection process in oil reservoirs.

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CHAPTER TWO

REVIEW OF PREVIOUS WORK

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CHAPTER TWO
REVIEW OF PREVIOUS WORK

2.1 Introduction

Steam injection is commonly divided into two process types steamflooding and steam soak. Figure 2.1 shows that when steam is injected into the reservoir an expanding "steam zone" is formed. The hot condensate leaving the steam zone and the cold water create a "water zone" ahead of the steam zone. The last flowing zone is the "oil zone". As steam is injected, heat is transferred to the surrounding reservoir formation and fluids by the two processes of conduction and convection.

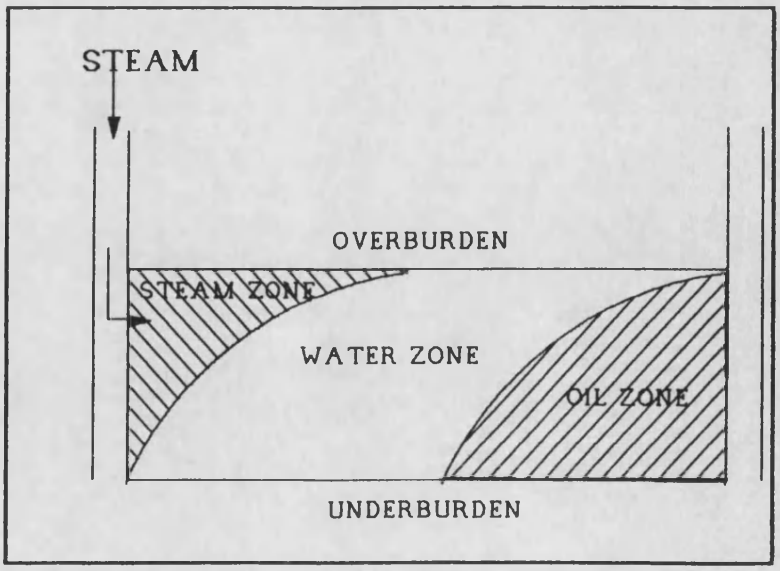

FIGURE 2.1 CROSS-SECTION OF STEAM INJECTION PROCESS

The literature includes a vast amount of theoretical work covering various aspects of oil recovery by steam. The modeling efforts differ in the level of complexity, from simple analytical models to numerical three-dimensional compositional model. The ultimate goal of all these models is the development of economical and a reliable engineering analysis tools to estimate oil recovery for a given mode of operation and to investigate alternative operating conditions to maximise oil recovery. The following selected models are useful way of tracing for representing the development stages of mathematical modelling of steam injection.

2.2 Selected Models

2.2.1 Analytical

2.2.1.1 Lauwerier

This model[1] is one of the initial attempt to analyse energy behavior during hot water injection. The model was based on the following assumptions:

- Linear incompressible flow in homogeneous sand.
- Constant fluid properties and saturations.
- Infinite thermal conductivity in the overburden

and underburden layers.

- Zero horizontal thermal conductivity in reservoir formation, overburden and underburden layers.
- Constant injection temperature.

Under the above assumptions Lauwerier obtain the following solution:

$$T = T_R + (T_F - T_R)\,erfc\frac{L_D}{2\sqrt{t_D - L_D}} \qquad \text{if } t_D > L_D \qquad (2.1a)$$

Else

$$T = T_R \qquad (2.1b)$$

Where:

T = temperature at any linear distance L, or radial r, from the injection well, °F

T_R = original reservoir temperature, °F

T_F = injection fluid temperature. °F

t_D = dimensionless time, calculated as follows

$$\frac{4k_{hob}M_{ob}t}{M_s^2 h_t^2} \qquad (2.1c)$$

L_D = dimensionless distance is given by

$$\frac{4(24)K_{hob}M_{ob}A}{5.615M_s\rho_F C_F q_F h_t} \qquad (2.1d)$$

$erfc$ = complementary error function, can be obtained from tables.

K_{hob} = thermal conductivity of the overburden, $Btu/hr-ft-°F$

M_{ob} = the heat capacity of the overburden, $Btu/ft^3-°F$

M_s = the heat capacity of the heated sand, $Btu/ft^3-°F$.

h_t = the gross thickness of the heated sand, ft.

A = cross section area of the sand, ft^2

C_F = specific heat of the injection fluid, $Btu/lb-°F$.

ρ_F = the density of the injection fluid, lb/ft^3.

q_F = hot fluid injection rate, Bbl/day

2.2.1.2 Marx and Langenheim Model

The works of Marx and Langenheim[2] were among the initial attempts to simulate steam injection process analytically. This model was first introduced in 1959. Their model is based upon the following assumptions:

- No gravity effect
- Homogeneous reservoir
- Constant injection rate
- No pressure drop in steam zone
- No hot water flow ahead of condensation front

Under the a bove assumptions, Marx and Langenheim developed the following equation:

$$A(t) = \frac{Q_t M_s h_t^2 f_1}{4K_{hob} M_{ob}(T_{st.} - T_R)} \qquad (2.2a)$$

Where:

$A(t)$= cumulative heated area at time t, ft^2

Qi= Heat injection rate, Btu / D

$f1$ = function of dimensiomnless time tD.

This term is presented as a table in their work:

$$e^{t_D} erfc\sqrt{t_D} + 2\sqrt{\frac{t_D}{\pi}} - 1 \qquad (2.2b)$$

2.2.2 Semianalytical

2.2.2.1 Rhee and Todd

This model[3] calculates oil recovery by Steamflooding with the consideration for steam distillation and gravity overlay effects . First the equations for the heat balance and the configuration of steam/liquid interface[4] are solved simultaneously to predict the volume of the overall heated zone and the volume and shape of the steam zone. second, the approximate enthalpy and material balances are combined with the thermodynamics of differential vaporization to derive a fully compositional method for calculating the volume and composition of steamdistilled hydrocarbons. finally, the results of the foregoing are combined with a

fluid flow model, similar to Higgins and Leighyon's[5] cell model for waterflood analysis to calculate oil recovery.

2.2.3 Numerical

2.2.3.1 Spillette and Nielson

Spillete and Nielson[6] solved the following system of partial differential equations simultaneously, through the use of "leap-frog" application of standard Alternating Direction Implicit methods for the solution if the mass-balance equation and the method of characteristics for the solution of the energy-balance equation.

Oil phase

$$\nabla \frac{k k_{ro} \rho_o}{\mu_o} (\nabla p_o - \gamma_o \nabla D) + q_o = \frac{\partial}{\partial t} (\phi \rho_o S_o) \qquad (2.3a)$$

Water phase

$$\nabla \frac{k k_{rw} \rho_w}{\mu_w} (\nabla p_w - \gamma_w \nabla D) + q_w = \frac{\partial}{\partial t} (\phi \rho_w S_w) \qquad (2.3b)$$

Energy Equation

$$\nabla K_{hR} \nabla T - \nabla \left(\rho_o C_o V_o - \rho_w C_w V_w \right) T + q_h =$$

$$\frac{\partial}{\partial t} \left(\phi \left(\rho_o S_o C_o + \rho_w S_w C_w \right) + (1 - \phi) \rho_f C_f \right) \qquad (2.3c)$$

The main assumptions are that no gas phase is present and the injected fluid reached thermal equilibrium instantaneously with the reservoir fluid and sand.

2.2.3.2 Shuter

Shuter introduced two simulators for steamflood process. The first simulator was three-phase linear simulator [7], where the second one was three-phase, two-dimensional [8]. Both simulators had fewer assumptions than any models previously written. His solution consisted of two steps, first, he solved fluid flow equations simultaneously by Newton iteration method then energy-balance was solved by alternating direction implicit procedure.

2.2.3.3 Coats K. H., George W. D. and Marcum B. E., and Coats

Coats and et al [9]. described a three-dimensional three-phase model for numerical simulation of steam

injection process. The main future of this model was the simultaneous solution of the mass and energy-balance and the elimination of the condensation term.

Coats[10] extended three-dimensional simulator to account for steam distillation, solution gas, and temperature dependent relative permeability. This model may considered as a step toward a fully compositional thermal model.

TABLE 2.1

NUMERICAL MODELS FOR HOT FLUID INJECTION

First author	Model used for: HWD: hot water drive SS: steam stimulation SD: steam drive	Number of dimensions	Number of phases	Number of constituents: Oil phase	Number of constituents: Gas phase	Type of medium: (homogeneous, heterogeneous, anisotropic)	Gravity	Capillary pressures	Oil distillation	Dissolved gases in oil	Compressibility of the rock	Relative permeabilities $k_r(\ )$ S: saturation	Oil viscosity $\mu_o(\ )$ j: j components	Gas viscosity $\mu_g(\ )$ j: j components	Oil density $\rho_o(\ )$
SPILLETTE [6]	HWD	2	2	1	0	Hetero	Yes	Yes	No	No	Yes	$k_r(S)$	$\mu_o(T)$	–	$\rho_o(T)$
SHUTLER [7]	SD	1 (1)	3	1	2	Homo	Yes	Yes	No	No	No	$k_r(S)$	$\mu_o(T, p)$	$\mu_g(T, p)$	$\rho_o(T, p)$
SHUTLER [8]	SD	2													
VINSOME [12]	HWD, SS, SD	3	3	1	1	Hetero	Yes	Yes	No	No	No	$k_r(S)$	$\mu_o(T)$	$\mu_g(T)$	$\rho_o(T, p)$
FERRER [13]	SD, SS	2 (2)	3	3	4	Homo	Yes	Yes	Yes	No	Yes	$k_r(S, T)$	$\mu_o(T)$	$\mu_g(T)$	$\rho_o(T, p)$
WEINSTEIN [14]	HWD, SS, etc.	1 (1)	3	2	2	Multi-layer	No	No	No	Yes	Yes	$k_r(S)$	$\mu_o(T, \text{GOR})$	$\mu_g(T, 2J)$	$\rho_o(T, p, \text{GOR})$
ABDALLA [15]	SD	2 (2)	3	1	1	Homo	No	Yes	No	No	No	$k_r(S)$	$\mu_o(T)$	$\mu_g(T)$	$\rho_o(T)$
COATS [9]	HWD, SD	3	3	1	1	Homo	Yes	Yes	No	No	Yes	$k_r(S)$	$\mu_o(T)$	$\mu_g(T)$	$\rho_o(T, p)$
COATS [16]	HWD, SS, SD	3	3	3	3	Aniso	Yes	Yes	Yes	Yes	Yes	$k_r(S, T)$	$\mu_o(T, 1J)$	$\mu_g(T, p, 2J)$	$\rho_o(T, p, 1J)$
COATS [10]	SD	3	3	2	2	?	Yes	Yes	Distillation or Dissolution		Yes	$k_r(S, T)$	$\mu_o(T, 1J)$	?	$\rho_o(T, p, 1J)$
LEMONNIER [17]	HWD, SS, SD	3	3	1	1	Aniso	Yes	Yes	No	No	Yes	$k_r(S, T)$	$\mu_o(T)$	$\mu_g(T)$	$\rho_o(T, p)$

(1) Two dimensional heat conduction.
(2) One dimensional heat flow in the z axis, to account for heat losses to the surrounding formations.

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CHAPTER THREE

FORMULATION OF THE DIFFERENTIAL EQUATIONS

CHAPTER THREE CONTENTS

FIGURES

CHAPTER THREE
FORMULATION OF THE DIFFERENTIAL EQUATIONS

3.1 Introduction

Before the reservoir can be modelled, it must be expressed in term of appropriate mathematical equation. To develop the mathematical equation the following points should be considered:

1. Governing equations.
2. Boundary condition.
3. Initial condition.

3.2 Governing equations

The governing Partial Differential Equations (PDE) of fluid flow and heat transfer through a porous medium are formulated by combining of the following equations[1, 2, 3] :

- equation of conservation
- Darcy's equation
- Fourier's equation
- an equation of state

3.2.1 Conservation of Mass :

Let us divid the reservoir into rectangular solid elements containing oil, water and gas phases. This configuration is shown in Figure 3.1. The mass flux changes within one of the rectangular solid elements (control volume) over time interval Δt can be written as [4, 5, 6] :

(mass in) - (mass out) + (mass source or sink) = (mass accumulation)

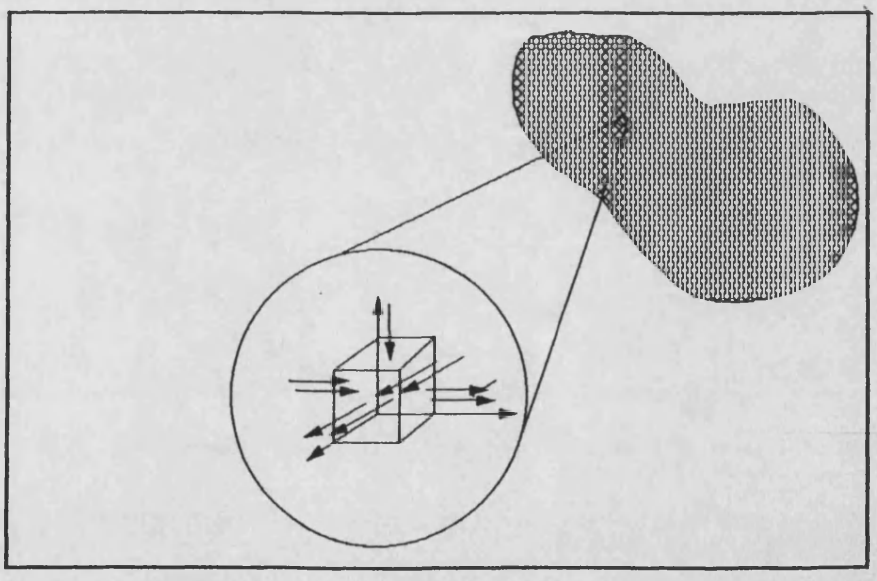

FIGURE 3.1 : F.D. LAY-OUT OF RESERVOIR

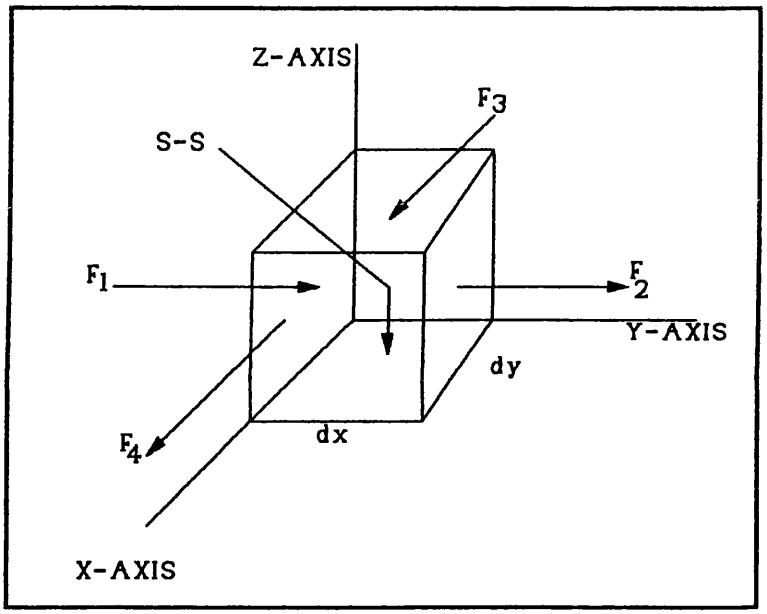

FIGURE 3.2 : VOLUME ELEMENT FOR THE FLUID FLOW EQUATION

Where:

F1 = Mass into the volume element, x-direction per unit time.

F2= Mass out the volume element, x-direction per unit time .

F3 = Mass into the volume element, y-direction per unit time.

F4 = Mass out the volume element, y-direction per unit time .

S-S = source and sink term per unit time.

X-direction :

let us first apply the mass balance equation to the x-direction. Referring to Figure 3.2 we can write the difference between the mass inflow and outflow across control volume over Δt as:

$$(F_1 \Delta y \Delta z \Delta t)_x - (F_2 \Delta y \Delta z \Delta t)_{x+\Delta x}$$

By express mass in terms of density and velocity we obtain the following equation:

$$(\rho v \Delta y \Delta z \Delta t)_x - (\rho v \Delta y \Delta z \Delta t)_{x+\Delta x} \qquad (3.1a)$$

Y-direction :

In a similar fashion to the x-direction we see from Figure 3.2 that the difference between the mass inflow and outflow in y-direction over Δt is

$$(F_3 \Delta x \Delta z \Delta t)_y - (F_4 \Delta x \Delta z \Delta t)_{y+\Delta y}$$

By replacing mass by density and velocity we get:

$$(\rho v \Delta x \Delta z \Delta t)_y - (\rho v \Delta x \Delta z \Delta t)_{y+\Delta y} \qquad (3.1b)$$

Source and sink :

Source and sink (S-S) term over Δt is

$$q_f \Delta x \Delta y \Delta z \qquad (3.1c)$$

Mass accumulation :

This term represents the net accumulation or depletion of fluid effect over Δt :

$$\left((\phi s \rho)_t - (\phi s \rho)_{t+\Delta t}\right) \Delta x \Delta y \Delta z \qquad (3.1d)$$

Where:

ρ = density, ft^3/lb

v = velocity, $ft/\sec$

q = production or injection, bbl/day

ϕ = porosity, $frc.$

s = saturation, $frc.$

Dividing all of the terms by $\Delta x \Delta y \Delta z \Delta t$ we obtain the following equation:

$$\frac{(\rho v)_x - (\rho v)_{x+\Delta x}}{\Delta x} + \frac{(\rho v)_y - (\rho v)_{y+\Delta y}}{\Delta y} + q = \frac{(\phi s \rho)_t - (\phi s \rho)_{t+\Delta t}}{\Delta t} \quad (3.2)$$

AS the limit of x, y, and t $\to 0$ we obtain the Partial Differential Equation (P.D.E) for single phase flow in the x and y directions:

$$\frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + q = \frac{\partial}{\partial t}(\phi s \rho) \quad (3.3)$$

3.2.2 Darcy's Equation:

Darcy's law[7, 8, 9] states that the velocity of a fluid in a porous medium is proportional to the pressure and inversely proportional to fluid viscosity. The expressions for the x and y velocity components for a single fluid phase are then :

$$v_x = -\frac{k_x}{\mu}\frac{\partial \Phi}{\partial x} \quad (3.4a)$$

$$v_y = -\frac{k_y}{\mu}\frac{\partial \Phi}{\partial y} \quad (3.4b)$$

Where:

k_x = permeability in x-direction

k_y = permeability in y-direction

μ = viscosity, cp

Multiphase Flow Equations:

Darcy's Equation (3.4) is modified to account for multiphase flow by introducing the relative permeability, so the multiphase velocity can be expressed in x or y directions as :

$$v_x = -\frac{k_x k_{rf}}{\mu_f}\frac{\partial \Phi}{\partial x} \qquad (3.5a)$$

$$v_y = -\frac{k_y k_{rf}}{\mu_f}\frac{\partial \Phi}{\partial y} \qquad (3.5b)$$

Where:

k_{rf} = Relative permeability of fluid (f)

μ_f = viscosity of fluid (f), cp

Substitution of the respective velocity terms into Equation (3.3) gives the P.D.E for multiphase flow in the x and y directions:

$$\frac{\partial}{\partial x}\left(\frac{k_x k_{rf}\rho_f}{\mu_f}\frac{\partial \Phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{k_y k_{rf}\rho_f}{\mu_f}\frac{\partial \Phi}{\partial y}\right)+q_f=\frac{\partial}{\partial t}(\phi s_f \rho_f) \qquad (3.6)$$

Where:

Φ = potential term, defined as:

$$P_f - \gamma_f D \qquad (3.7)$$

P_f = fluid pressure, psi

γ_f = fluid specific weight, psi/ft

D = Reservoir depth, ft

Oil phase:

Equation (3.6) can be written for the oil phase as:

$$\frac{\partial}{\partial x}\left(\frac{k_x k_{ro}\rho_o}{\mu_o}\frac{\partial \Phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{k_y k_{ro}\rho_o}{\mu_o}\frac{\partial \Phi}{\partial y}\right)+q_o=\frac{\partial}{\partial t}(\phi s_o \rho_o) \qquad (3.8a)$$

Water phase:

Similarly, the equation for the water phase is:

$$\frac{\partial}{\partial x}\left(\frac{k_x k_{rw}\rho_w}{\mu_w}\frac{\partial \Phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{k_y k_{rw}\rho_w}{\mu_w}\frac{\partial \Phi}{\partial y}\right)+q_w=\frac{\partial}{\partial t}(\phi s_w \rho_w) \qquad (3.8b)$$

Gas phase:

Different situation exists for the gas phase, since the gas can flow freely, e.g. as steam or gas in saturated Reservoir, On the other hand, gas may dissolve in the oil or water in the case of undersaturated reservoir. To account for these situations, the following Partial Differential Equation is used:

$$\frac{\partial}{\partial x}\left(\frac{k_x k_{rg}\rho_g}{\mu_g}\frac{\partial \Phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{k_y k_{rg}\rho_g}{\mu_g}\frac{\partial \Phi}{\partial y}\right)+$$
$$\frac{\partial}{\partial x}\left(\frac{Rs_o k_x k_{ro}\rho_o}{\mu_o}\frac{\partial \Phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{Rs_o k_y k_{ro}\rho_o}{\mu_o}\frac{\partial \Phi}{\partial y}\right)+$$
$$\frac{\partial}{\partial x}\left(\frac{Rs_w k_x k_{rw}\rho_w}{\mu_w}\frac{\partial \Phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{Rs_w k_y k_{rw}\rho_w}{\mu_w}\frac{\partial \Phi}{\partial y}\right)+$$
$$q_g + Rs_o q_o + Rs_w q_w = \frac{\partial}{\partial t}(\phi s_g \rho_g + \phi Rs_o s_o \rho_o + \phi Rs_w s_w \rho_w) \qquad (3.8c)$$

Where:

Rs_o, Rs_w = are oil and water gas solution ratios, scf/STB

In the case of steam injection, if we assume that Rs values are zero, in which case Equation (3.8c) reduces to:

$$\frac{\partial}{\partial x}\left(\frac{k_x k_{rg}\rho_g}{\mu_g}\frac{\partial \Phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{k_y k_{rg}\rho_g}{\mu_g}\frac{\partial \Phi}{\partial y}\right)+q_g=\frac{\partial}{\partial t}(\phi s_g \rho_g) \qquad (3.9)$$

3.2.3 Energy Equation:

The energy Partial Differential Equations can be derived in a similar way to that for fluid flow[10,11]. The heat flux on a volume element taken inside the porous medium, Figure 3.3, over Δt can be written as:

(heatin by conduction) - (heatout by conduction) +
(heatin by convection) - (heatout by convection) +
(heat source or sink) = (heat accumulation)

Convection term:

The convection term over Δt is calculated as follows:

$$\{(\rho_o v_{ox} H_o + \rho_w v_{wx} H_w + \rho_g v_{gx} H_g)_x \Delta y \Delta z \Delta t -$$
$$(\rho_o v_{ox} H_o + \rho_w v_{wx} H_w + \rho_g v_{gx} H_g)_{x+\Delta x} \Delta y \Delta z \Delta t\} +$$
$$\{(\rho_o v_{oy} H_o + \rho_w v_{wy} H_w + \rho_g v_{gy} H_g)_y \Delta x \Delta z \Delta t -$$
$$(\rho_o v_{oy} H_o + \rho_w v_{wy} H_w + \rho_g v_{gy} H_g)_{y+\Delta y} \Delta x \Delta z \Delta t\} \qquad (3.10c)$$

Where:

H_f = enthalpy of fluid f, Btu

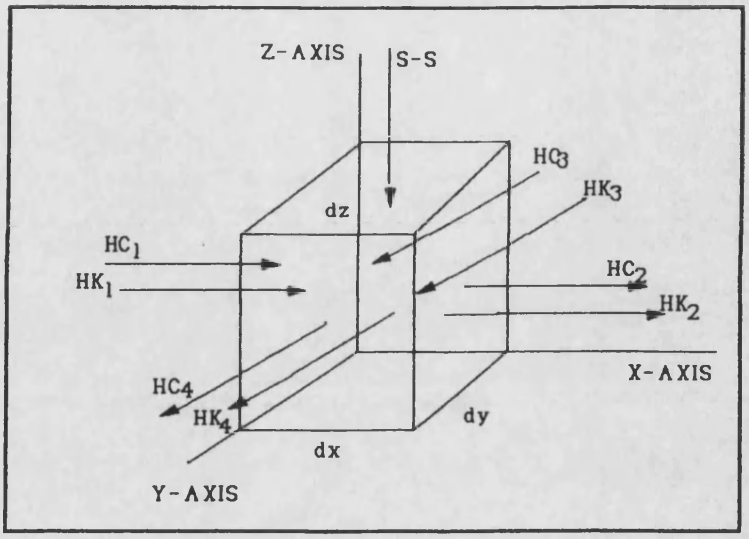

FIGURE 3.3 : VOLUME ELEMENT FOR THE ENERGY FLOW EQUATION

Where:

HK1	=	Heat Conduction into the volume element, x-direction per unit time
HK2	=	Heat Conduction out the volume element, x-direction per unit time
HK3	=	Heat Conduction into the volume element, y-direction per unit time
HK4	=	Heat Conduction out the volume element, y-direction per unit time
HC1	=	Heat Convection into the volume element, x-direction per unit time
HC2	=	Heat Convection out the volume element, x-direction per unit time
HC3	=	Heat Convection into the volume element, y-direction per unit time
HC4	=	Heat Convection out the volume element, y-direction per unit time
S-S	=	source and sink term per unit time

Conduction Term:

Referring to Figure 3.3 we see that the difference of heat conducted in and out of the control volume element in all three directions (x, y and z) over Δt can be expressed as:

$$\left\{(Q_{cond}\Delta y\Delta z\Delta t)_x-(Q_{cond}\Delta y\Delta z\Delta t)_{x+\Delta x}\right\}+$$
$$\left\{(Q_{cond}\Delta x\Delta z\Delta t)_y-(Q_{cond}\Delta x\Delta z\Delta t)_{y+\Delta y}\right\}+$$
$$\left\{(Q_{cond}\Delta y\Delta x\Delta t)_z-(Q_{cond}\Delta y\Delta x\Delta t)_{z+\Delta z}\right\} \qquad (3.10a)$$

Where:

Q_{cond} = heat flow, Btu/day, which calculated as

$$-K_h A\Delta T \qquad (3.10b)$$

K_h = thermal conductivity, Btu/hr-ft-F

T = temperature, $^\circ F$

Source and Sink term:

The equation for the source and sink term is:

$$q_{ht}\Delta x\Delta y\Delta z\Delta t \qquad (3.10d)$$

Where:

q_{ht} = heat generated within the element, Btu/day

Heat Accumulation Term:

Heat Accumulation is the finial term to be calculated over the time interval Δt:

$$\Delta x \Delta y \Delta z \{\phi T (S_o \rho_o C_{po} + S_w \rho_w C_{pw} + S_g \rho_g C_{pg}) + (1-\phi) T \rho_R C_{pR}\}_{t+\Delta t} -$$
$$\Delta x \Delta y \Delta z \{\phi T (S_o \rho_o C_{po} + S_w \rho_w C_{pw} + S_g \rho_g C_{pg}) + (1-\phi) T \rho_R C_{pR}\}_{t} \qquad (3.10d)$$

Where:

c_{pf} = specific heat for fluid or formation (f), Btu/lb-F

Dividing Equations (3.10) by $\Delta x \Delta y \Delta z \Delta t$ and taken the limit as $\Delta x \Delta y \Delta z \Delta t \to 0$ we have the energy P.D.E:

$$\frac{\partial}{\partial x}\left(K_{hx} A_x \partial \frac{T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{hy} A_y \partial \frac{T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{hz} A_z \partial \frac{T}{\partial z}\right) -$$
$$\frac{\partial}{\partial x}(\rho_o H_o v_{ox} + \rho_w H_w v_{wx} + \rho_g H_g v_{gx}) - \frac{\partial}{\partial y}(\rho_o H_o v_{oy} + \rho_w H_w v_{wy} + \rho_g H_g v_{gy}) + q_{ht} =$$
$$\frac{\partial}{\partial t}\{\phi T (S_o \rho_o c_{po} + S_w \rho_w c_{pw} + S_g \rho_g c_{pg}) + (1-\phi) T \rho_R c_{pR}\} \qquad (3.11)$$

3.3 Initial and Boundary Conditions:

The mathematical model developed is not complete without the description of a reservoir initial and boundary conditions.

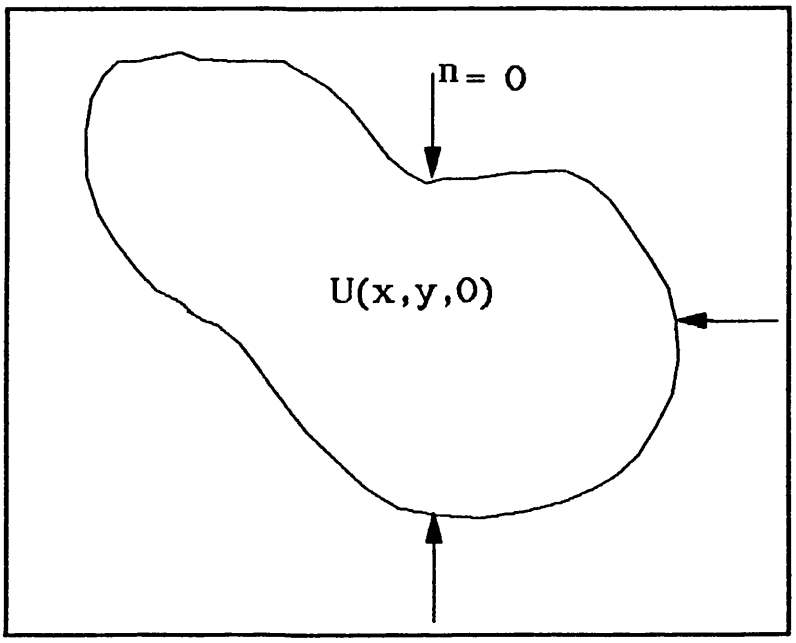

FIGURE 3.4 INITIAL AND BOUNDARY CONDITIONS

The initial conditions of the system are given by:

$$U(x,y,0) = U_{initial}$$

Where U could be pressure, temperature, fluid properties or rock properties

A No-flow or closed boundary type situation is assumed. This type of boundary is simply expressed as :

$$\partial \frac{P}{\partial \eta} \text{ and } \partial \frac{T}{\partial \eta} = 0 \qquad (3.12)$$

Where :

η = normal vector to the confining boundary

Fluid influx across the reservoir limits may be replaced by injection or production rates of wells long the edge of the reservoir domain.

The solution of the developed partial differential equations of flow and energy gives the pressure, saturation and temperature distribution throughout the reservoir.

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CHAPTER FOUR

SOLUTION METHOD

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SOLUTION METHOD

4.1 Introduction

The equations which describe the flow of fluids and energy in porous media were derived in chapter 3. These equations are nonlinear partial differential equations, which are too complicated to by amenable to analytical solution.

In general, numerical method is only way to solve this type of equations. A number of numerical techniques are adequate to solve nonlinear partial differential quations. The most widely used in reservoir modeling is the Finite Difference Method (FDM).

The aim of finite of difference method is to transform a given continuum problem into a discrete mathematical model. Because the FDM requires only arithmetic operation of addition, subtraction, multiplication, and division that makes it suitable for solution on digital computer.

4.2 Basic Equations And Assumptions

Before the transformation of the partial differential equations to FDM, it is necessary to introduce the following equations and assumptions :

4.2.1 Basic Equations

the saturation equation

$$S_o + S_w + S_g = 1 \qquad (4.1)$$

and the capillary pressure equations

$$P_{cwo} = P_o - P_w \qquad (4.2a)$$

$$P_{cgo} = P_o + P_g \qquad (4.2b)$$

Where:

Pcwo = capillary pressure of water-oil system, psia

Pcgo= capillary pressure of gas-oil system, psia

4.2.2 Assumptions

1. One or two dimensional model Cartesian co-ordinates)

2. Three-phases (oil , water and gas)

3. Capillary pressure and gravity are considered.

4. IMPES method is used to solve fluid flow equation

5. Solution of the energy equation assumes that there is no solution of gas in the oil and water phases. This restriction does not apply to isothermal option.

6. Fluid properties are function of pressure and temperature.

7. Five-point space discretisation is considered in finite difference solution, Figure 4.1.

8. No-flow boundary.

9. Centre block grid configuration is used.

10. Uniform or nonuniform grid spacing is permitted.

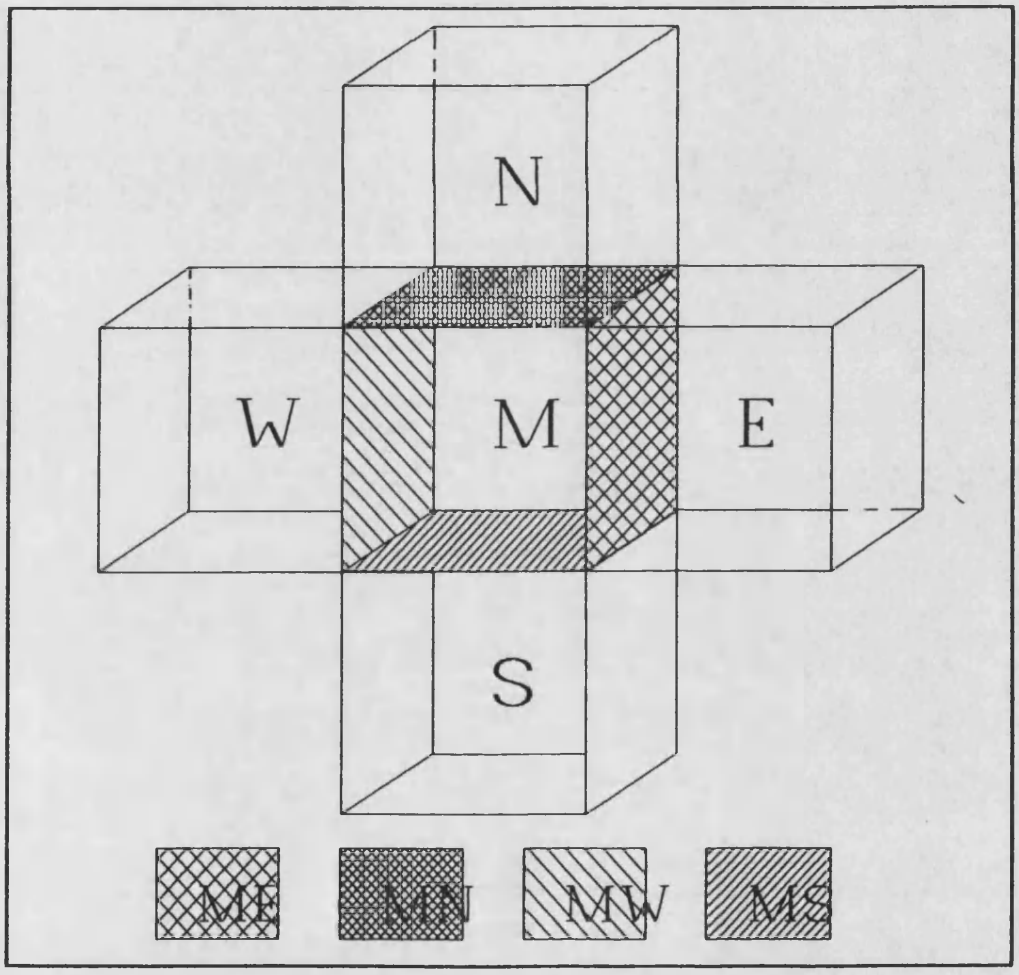

FIGURE 4.1: DIFFERENCE MODEL GRID SYSTEM

4.3 Finite Difference Approximation

To transform the second order derivatives in the flow and energy equations into finite difference form, the central-difference approximation is used.[1,2]

For example,the pressure derviative in the P.D.E for the oil phase is expressed in F.D.E form as follows [3, 4, 5] :

$$\frac{\partial}{\partial x}\left(\frac{k_x k_{ro}}{\mu_o B_o}\left(\frac{\partial P_o}{\partial x}-\frac{\gamma_o \partial Z}{\partial x}\right)\right)=\Delta T_o\left(\Delta P_o-\gamma_o \Delta Z\right) \qquad (4.3a)$$

The difference operations Δ can be expanded further to:

$$T^n_{oMW}\left(P_{oW}-P_{oM}\right)^{n+1}+T^n_{oME}\left(P_{oE}-P_{oM}\right)^{n+1}+$$
$$T^n_{oMW}\gamma_o\left(Z_W-Z_M\right)^{n+1}+T^n_{oME}\gamma_o\left(Z_E-Z_M\right)^{n+1} \qquad (4.3b)$$

Where the finite-difference transmissibility T ,for example, for oil at the grid boundary MW is given by

$$T^n_{oMW}=GC_{MW}F^*_1oF^*_2o \qquad (4.4a)$$

Where :

GCMW = the Geometric Constant and it is calculated as:

$$\frac{2A_MA_WK_MK_W}{A_MK_MX_E+A_EK_EX_M} \qquad (4.4b)$$

$$F^*_1o=\frac{1}{\mu_oB_o} \qquad (4.4c)$$

$$F_{2o}^{+} = K_{ro} \qquad (4.4d)$$

+ indicates upstream value.

Where:

A= cross-sectional area of a grid, ft^2.

K= Absloute permeability, darcy.

X= distance, ft.

The following " upstream" conception is used to evaluate F_{1o} and F_{2o} for flow between M and W as follows:

if $\Phi_{oM}^{n} \geq \Phi_{oW}^{n}$ then

$$F_{1o}^{+} = f(P_{oM})$$
$$F_{2o}^{+} = f(S_{oM}) \qquad (4.4e)$$

if $\Phi_{oM}^{n} \leq \Phi_{oW}^{n}$ then

$$F_{1o}^{+} = f(P_{oW})$$
$$F_{2o}^{+} = f(S_{oW}) \qquad (4.4f)$$

A similar procedure is employed for all other phases and grids.

The first order time derivatives are approximated by the backward difference, for example, the time derivative of fluid flow equation is written as:

$$\frac{\partial}{\partial t}\frac{\phi S_o}{B_o} = \frac{1}{\Delta t}\Delta_t\left(\frac{\phi S_o}{B_o}\right) \qquad (4.5a)$$

Where :

$$\Delta_t\left(\frac{\phi S_o}{B_o}\right) = \left(\frac{\phi S_o}{B_o}\right)^{n+1} - \left(\frac{\phi S_o}{B_o}\right)^{n} \qquad (4.5b)$$

By applying these approximations to all phases and direction we can generate a complete finite difference procedure which describes the flow of fluid and heat in the reservoir .

4.4 Solution of Fluid flow Equation

In this section, we develop the finite difference scheme necessary to solve the fluid flow equations, first by transforming the fluid flow partial differential Equations (3.8) into the following F.D.E:

A- oil phase:

$$\Delta T_o\left(\Delta P_o - \gamma_o \Delta Z\right) + q_o = \frac{V_b}{\Delta t}\Delta_t\left(\frac{\phi S_o}{B_o}\right) \qquad (4.6a)$$

B- Water Phase:

$$\Delta T_w\left(\Delta P_w - \gamma_w \Delta Z\right) + q_w = \frac{V_b}{\Delta t}\Delta_t\left(\frac{\phi S_w}{B_w}\right) \qquad (4.6b)$$

C- Gas Phase:

$$\Delta T_g(\Delta P_g - \gamma_g \Delta Z) + \Delta T_o R_{so}(\Delta P_o - \gamma_o \Delta Z) +$$
$$\Delta T_w R_{sw}(\Delta P_w - \gamma_w \Delta Z) + (q_g + g_o Rso + g_w Rsw) =$$
$$\frac{V_b}{\Delta t}\Delta_t\left(\frac{\phi S_g}{B_g} + \frac{\phi S_o R_{so}}{B_o} + \frac{\phi S_w R_{sw}}{B_w}\right)$$

(4.6c)

Secondly the IMPES, implicit pressure explicit saturation, method will be used to solve the fluid flow equation, first for the pressure and then the saturation distributions .

4.4.1 IMPES Method

In general, reservoir simulators use one of the three solution methods: Implicit Pressure Explicit Saturation (IMPES), Simultaneous Solution (SS), or SEQuenstial method (SEQ).

The choice of the most efficient method depends of the computer availability of:

A- computer work.

B- computer storage.

A- Computer Work

The total computer work if a method is given by[3,6]:

$$W_T = W_{st} TT_{st}$$

Where:

W_T = Total computer Work.

TT_{st} = Total number of Time STeps.

W_{st} = Computer Work by time STep.

The work of one time step, for direct elimination method, is estimated as follows:

$$W_{st} = CB^2N$$

Where:

C = A constant depends on computer type and programme efficiency.

B = A matrix Band-width.

N = Number of simultaneous equations or unknown variables.

For one dimensional simulator the computer work per one time step is reduce to :

$$W_{st} = CN^3$$

The computer work per time step required for 2-dimensional, 3-phase fluid flow problems are given in table 4.1 for the three solution methods.

TABLE 4.1 COMPARISON OF STEP WORK FOR 2-D, 3-PHASE FLOW				
Method	Band-width B	No. of un-known N	Work/timestep Wst	Wst/Wst(IMPES)
IMPES	J	IJ	CIJ^3	1
SEQ	J,2J	IJ,2IJ	$C(IJ^3+8IJ^3)$	9
SS	3J	3IJ	$27CIJ^3$	27

A- Computer Storage

The computer storage required for 2-dimensional, 3-phase simulator is estimated by:

$$S = ND \times N \times B$$

Where :

S = Computer Storage.

ND = Number of Dimension.

Table 4.2 shows the approximate storage required for 2-dimension and 3-phase flow problem.

TABLE 4.2 COMPARISON OF STORAGE REQUIREMENT FOR 2-D, 3-PHASE FLOW				
Method	Band-width B	No. of un-known N	Storage S	S/S(IMPES)
IMPES	J	IJ	$2CIJ^2$	1
SEQ	J,2J	IJ,2IJ	$8CIJ^2$	4
SS	3J	3IJ	$18CIJ^2$	9

The comparison of the the methods, as far as computer work and storage requirements, shows that the demand for work and storage increases dramatically as we go from IMPES to SS methods.

4.4.1.1 Implicit Calculation of pressure

The IMPES method utilises capillary pressure and saturation equations to combine the three single-phase Equations (4.6) into the following multiphase equation[7, 8, 9] . The derivation of the IMPES equation is explained in Appendix A.

$$(B_o - B_g R_{so})^{n+1}(\Delta T_o(\Delta P_o - \gamma_o \Delta Z) + q_o) +$$

$$(B_w - B_g R_{sw})^{n+1}(\Delta T_w(\Delta P_w - \Delta P_{cw} - \gamma_w \Delta Z) + q_w) +$$

$$B_g^{n+1}(\Delta T_g(\Delta P_g + \Delta P_{cg} - \gamma_g \Delta Z) + \Delta T_o R_{so}(\Delta P_o - \gamma_o \Delta Z) +$$

$$\Delta T_w R_{sw}(\Delta P_w - \gamma_w \Delta Z) + (q_g + g_o Rso + g_w Rsw) =$$

$$\frac{V_b}{\Delta t}\{(\phi B_o)^{n+1}(S_o)^n\left(\frac{1}{B_o}\right)' + \left(\frac{\phi B_g}{B_o}\right)^{n+1}(S_o)^n(R_{so})' +$$

$$(\phi B_w)^{n+1}(S_w)^n\left(\frac{1}{B_w}\right)' + \left(\frac{\phi B_g}{B_w}\right)^{n+1}(S_w)^n(R_{sw})' +$$

$$(\phi B_g)^{n+1}(S_g)^n\left(\frac{1}{B_g}\right)' + (\phi)'\}\Delta_t P_o +$$

$$\left\{(\phi B_w)^{n+1}(S_w)^n\left(\frac{1}{B_w}\right)' + \left(\frac{\phi B_g}{B_w}\right)^{n+1}(S_w)^n(R_{sw})' + S_w(\phi)'\right\}\Delta_t P^n_{cwo} +$$

$$\left\{(\phi B_g)^{n+1}(S_g)^n\left(\frac{1}{B_g}\right)' + S_g(\phi)'\right\}\Delta_t P_{cgo} \qquad (4.7)$$

Where:

$\left(\frac{1}{B_f}\right)'$ = gradient of reciprocal of the formation volume factor

$$= \frac{d\left(\frac{1}{B_f}\right)}{dP_f}$$

$(\phi)'$ = porosity gradient

$$= \frac{d\phi}{dp}$$

$(R_f)'$ = gradient of solution gas of phase (f)

$$= \frac{d(R_f)}{dp_f}$$

$$\Delta_t P = P^{n+1} - P^n$$

If the IMPES equation is written for all grids in the x-y directions , using the standard ordering notation, the following form of linear equation is obtained:

$$e_M P_W^{n+1} + a_M P_N^{n+1} + b_M P_M^{n+1} + c_M P_S^{n+1} + f_M P_E^{n+1} = d_M \qquad (4.8)$$

4.4.1.2 Explicit calculation of Saturation

After the new pressures are calculated, the new saturations are computed explicitly as follows:

Oil saturation

$$S_o^{n+1} = \frac{\Delta t B_o^{n+1}}{\phi V_b}\left(\Delta T_o\left(\Delta P_o - \gamma_o \Delta Z\right) + q_o\right) +$$

$$\left(\frac{S_o^n}{\phi}(\phi)' + \frac{(S_o)^n}{B_o}\left(\frac{1}{B_o}\right)'\right)\Delta P + S_o^n \qquad (4.9a)$$

Water saturation

$$S_w^{n+1} = \frac{\Delta t B_w^{n+1}}{\phi V_b}\left(\Delta T_w\left(\Delta P_o - \Delta P_{cwo} - \gamma_w \Delta Z\right) + q_w\right) +$$

$$\left(\frac{S_w^n}{\phi}(\phi)' + \frac{(S_w)^n}{B_w}\left(\frac{1}{B_w}\right)'\right)\Delta P + S_o^n \qquad (4.9b)$$

Gas saturation

The gas saturation is calculated using Equation (4.1)

$$S_g^{n+1} = 1 - S_o^{n+1} - S_w^{n+1} \qquad (4.9c)$$

4.5 Solution of Energy Equation

The partial differential equation for energy conservation is apprroximated in finite difference form[10,11,12,13] as:

$$\Delta T_o H_o(\Delta P_o - \gamma_o \Delta Z) + \Delta T_w H_w(\Delta P_w - \gamma_w \Delta Z) + \Delta T_g H_g(\Delta P_g - \gamma_g \Delta Z) +$$
$$\Delta(K_h A \Delta T)_x + \Delta(K_h A \Delta T)_y + \Delta(K_h A \Delta T)_z + q_{ht} - q_{hl} =$$
$$\frac{V_b}{\Delta t}\Delta_t\{(\phi S_o \rho_o H_o + \phi S_w \rho_w H_w + \phi S_g \rho_g H_g) + (1-\phi)\rho_r C_r T\}$$

(4.10a)

Where:

H_f = enthalpy , subscript f stand for phase or system, Btu

K_h = thermal conductivity, $Btu/hr-ft-°F$

$q_{ht} = heat source \vee sink$

$q_{hl} = heat loss$

C_f - Specific heat capacity, subscript f stand for phase or system.

A similar form to Equation (4.8) can be produced for the heat finite difference equation.

$$e_M T_V^{n+1} + a_M T_N^{n+1} + b_M T_M^{n+1} + c_M T_S^{n+1} + f_M T_E^{n+1} = d_M \qquad (4.11)$$

4.6 treatment of production and injection term:

In actual reservoir practice a well is not only a source and sink term,it is also where all observations and measurements are made. So the way inwhich the simulator treats the well situation has a great impact on the accuracy of the simulation results[14,15].

Explicit treatment for the well is used. One of the following parameters for the new time level must be specified:

A-. constant well rate for phase f.

B-. Total fluid rates

C-. wellbore pressure.

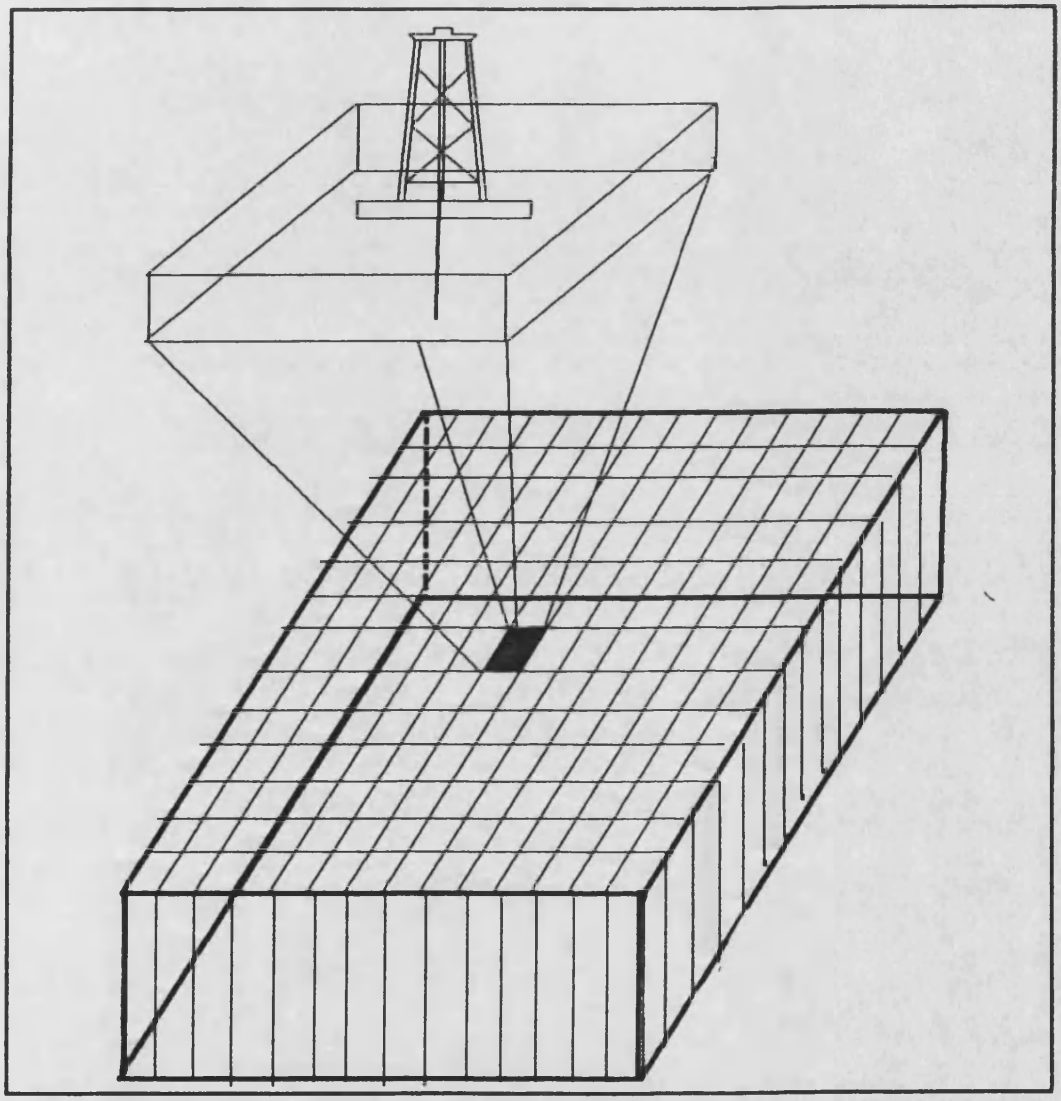

FIGURE 4.2 WELL PRESENTATION IN GRID SYSTEM

A- Constant well rate:

In this case one of the phases is given at the new time as:

$$q_f^{n+1} = \text{constant} \qquad (4.12a)$$

Then mobility ratio terms are used to calculate the other phase flow rates. For example, if oil rate is specified in Equation (4.12a) then the water and gas rates are calculated as :

$$q_w^{n+1} = F_{\lambda wo}\, q_o^{n+1} \qquad (4.12b)$$

$$q_g^{n+1} = F_{\lambda go}\, q_o^{n+1} + R_o^{n+1} q_o^{n+1} + R_w^{n+1} q_w^{n+1} \qquad (4.12c)$$

Where:

$F_{\lambda wo}$ = mobility ratio of water to oil

$$= \frac{\lambda_w}{\lambda_o} \qquad (4.12d)$$

$F_{\lambda go}$ = mobility ratio of gas to oil

$$= \frac{\lambda_g}{\lambda_o} \qquad (4.12e)$$

$$\lambda_f = \frac{K_{rf}}{\mu_f B_f} \qquad (4.12f)$$

B- Total fluid rate:

Total fluid rates can be expressed mathematically as:

$$q_t = q_o + q_w + q_g \qquad (4.13a)$$

For the new time level the total fluid rates should be specified:

$$q_t^{n+1} = \text{constant} \qquad (4.13b)$$

Each phase is then calculated by the following equations:

Oil rate:

$$q_o^{n+1} = \left(\frac{\lambda_o}{\Lambda}\right) q_t^{n+1} \qquad (4.13c)$$

Water rate:

Equation (4.12d) is then used to calculate the water rate.

Gas rate:

The gas rate can then be determined by subtraction:

$$q_g = q_t - q_o - q_w \qquad (4.13d)$$

Where:

Λ = total mobility of all producing fluids, darcy / cp.

$$= \lambda_o + \lambda_w + \lambda_g \qquad (4.13e)$$

C- Constant wellbore presssure[16]:

The configuration of a well producing from a circular drainage area with radius (r_e) as shown in figure (2.3) is

usually used for calculating well production, as following:

$$q_o = \frac{7.07\lambda_o K_x K_y h(\bar{p} - p_{wf})}{\ln(r_e/r_w) - 0.75 + S} \qquad (4.14a)$$

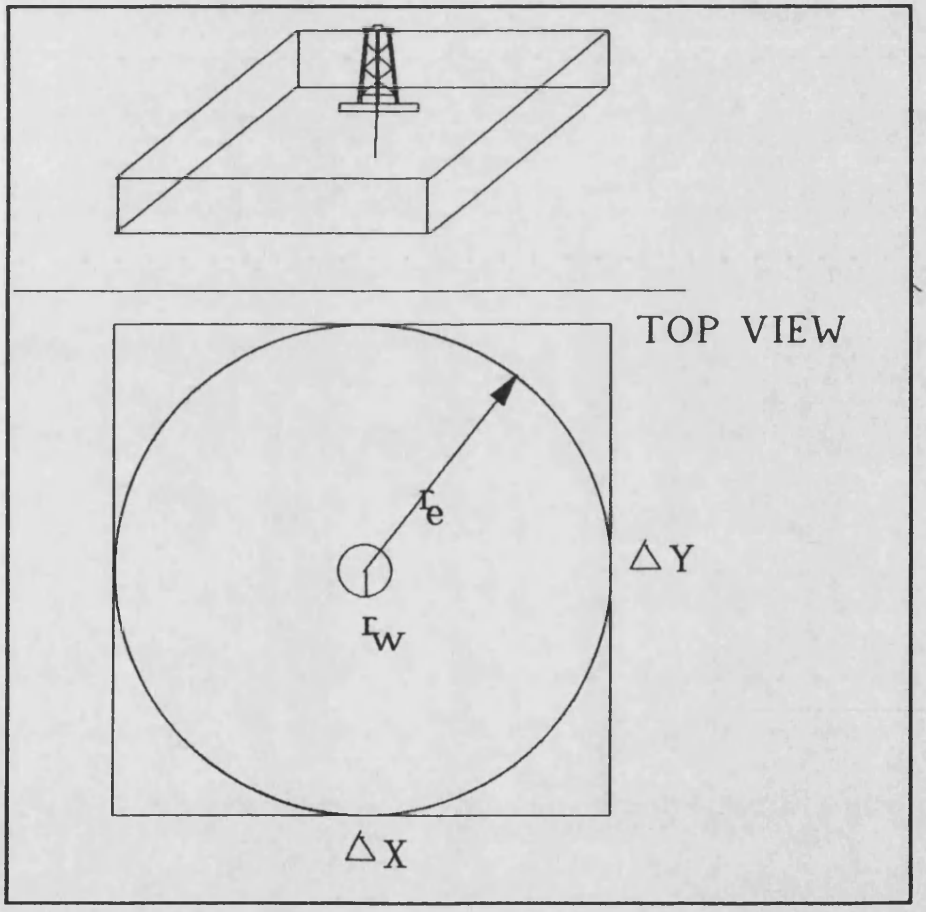

FIGURE 4.3 IMPOSING OF GRID AREA ON WELL DRAINAGE AREA

The grids used in the simulator however, are not circular but rectangular or polygonal in shape, as shown in Figure 4.2. The equivalent of r_e, in terms of the grid dimensions Δx and Δy, should be used Equation (4.14a) to calcualte production. This is:

$$r_e \approx \sqrt{\frac{\Delta x \Delta y}{\pi}} \qquad (4.14b)$$

Where:

r_w = Well radius, ft

r_e = drainage radius, ft

$\overline{p}$ = average grid pressure, psi

and

S = skin effect

Injection or sink term:

During isothermal injection we only need to consider one fluid phase, and consequently the other two phases are get to be zero. However, in steam injection, steam and water need to be considered. The injection rate of each phase is expressed as:

$$q_g = q_t x \qquad (4.15a)$$

$$q_w = q_t(1 - x) \qquad (4.15b)$$

Heat gain from the above qg and qw is calculated as:

$$(q_g C_{pg} \rho_g + q_w C_{pw} \rho_w) T_i^{n+1} \qquad (4.15c)$$

Where:

x = steam quality, frc.

T_i = injection temperature °F

4.7 Heat Loss

4.7.1 Heat loss by Production:

Fluid acts as a heat carrier in convective transport through the porous medium. Therefore, the production of fluids also create heat loss.
This loss can be calculated as:

$$q_{hl} = (q_o \rho_o C_{po} + q_w \rho_w C_{pw} + q_g \rho_g C_{pg}) T^{n+1} \qquad (4.16)$$

Where:

T = well or grid temperature, °F

4.7.2 Heat loss to the surrounding formation:

As it shown in Figure 4.4, the oil reservoir is surrounded by two impermeable formations (overburden and underburden). Heat tarnsfer to these adjacent formations occurs by vertical conduction. This conduction heat transport is calculated as:

$$K_{hob} \frac{\partial^2 T}{\partial Z^2} = \rho_{ob} C_{pob} \frac{\partial T}{\partial t} \qquad (4.17)$$

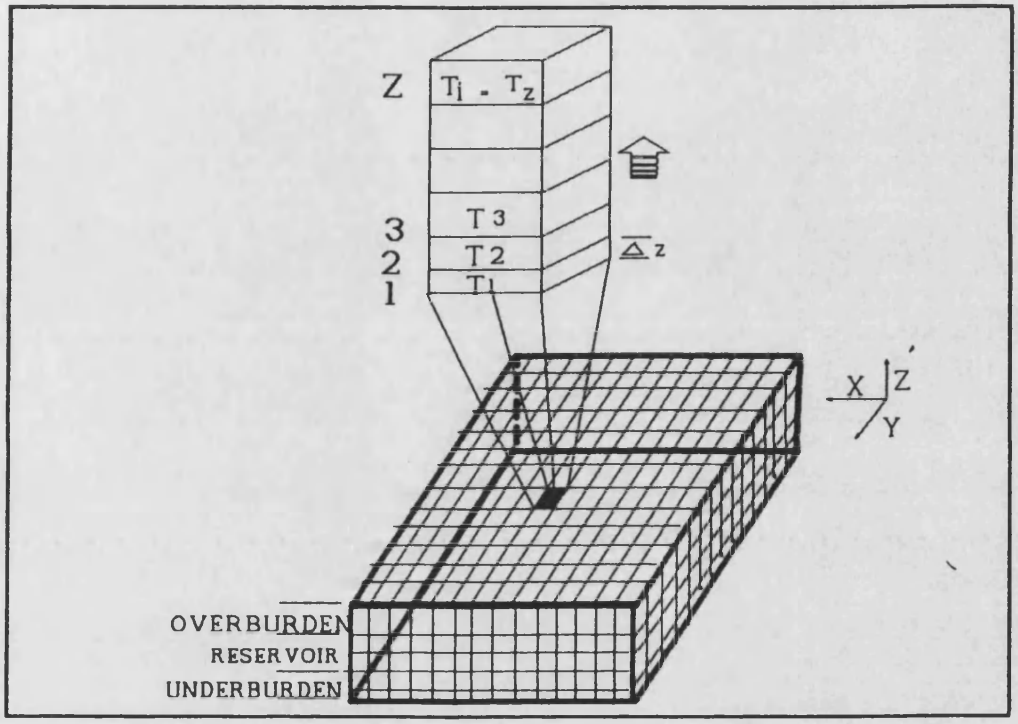

FIGURE 4.4 HEAT LOSS TO THE OVERBURDEN AND UNDERBURDEN

As shown in Figure 4.4, the overburden formation is divided into rectangular grids with uniform thickness (grids closer to the reservoir have a smaller thickness). Equation (4.17) is represented in implicit difference form as:

$$AT^{n+1}_{i,k-1} + BT^{n+1}_{i,k} + CT^{n+1}_{i,k+1} = D \qquad (4.18)$$

Where:

$$A = \frac{2\sigma}{\Delta Z_{i,k}(\Delta Z_{i,k} + \Delta Z_{i,k-1})}$$

$$C = \frac{2\sigma}{\Delta Z_{i,k}(\Delta Z_{i,k} + \Delta Z_{i,k+1})}$$

$$D = -\frac{T^{n}_{i,K}}{\Delta t}$$

$$B = -\left(A + C + \frac{1}{\Delta t}\right)$$

$$\sigma = \frac{K_{hob}}{\rho_{ob} C_{pob}}$$

The boundary conditions for this system are:

$$T_z(x,y,Nz,t) = T_t$$
$$T_1(x,y,0,t) = T_r$$

Equation (4.18) produces a tridiagonal matrix which can be solved by using the Thomas Algorithm. The solution is then used to obtain Heat loss.

$$q_{hl} = -2K_{hob}\Delta x \Delta y(T_r - T_1)$$

4.8 Automatic time step control:

Todd, O'Dell and Hiraasaki[17] introduced this concept in (1972). The approach is to select the largest stable Δt so that the maximum saturation or pressure do not exceed there limits ΔS_{LIM} and ΔP_{LIM}. The automatic time-step contol calculated as:

$$\Delta t^{n+1} = \min\left\{\Delta t_S, \Delta t_P\right\} \qquad (4.19a)$$

Where :

$$\Delta t_S = \Delta t^n \frac{\Delta S_{LIM}}{\Delta S_{MAX}} \qquad (4.19b)$$

$$\Delta t_P = \Delta t^n \frac{\Delta P_{LIM}}{\Delta P_{MAX}} \qquad (4.19c)$$

ΔS_{LIM} and ΔP_{LIM} are problem dependents

$$\Delta S_{MAX} = MAX_{\text{over all grids}}\left\{S^n - S^{n-1}\right\} \qquad (4.19d)$$

$$\Delta P_{MAX} = MAX_{\text{over all grids}}\left\{P^n - P^{n-1}\right\} \qquad (4.19d)$$

4.9 Solution of the Simultaneous Equations:

The matrix form of Equation (4.20) is generated by writing F.D.E's of fluid or energy for all grids in the x-y system.

$$\bar{A}u = d \qquad (4.20)$$

Where $\bar{A}$ is the coefficient matrix with pentadiagonal form. The vectors u and d each have M elements. The vecoter u contains the unknown functional values, e.g pressure as the case of the fluid flow equations or temperature in the energy equations.

$$\overline{A} = \begin{pmatrix} b_1 & c_1 & & f_1 & & & & & \\ a_2 & b_2 & c_2 & & f_2 & & & & \\ & \cdot & \cdot & \cdot & & \cdot & & & \\ e_4 & & a_4 & b_4 & c_4 & & f_4 & & \\ & \cdot & & \cdot & \cdot & \cdot & & \cdot & \\ & & \cdot & & \cdot & \cdot & \cdot & & \cdot \\ & & & \cdot & & \cdot & \cdot & \cdot & \\ & & & & e_{M-1} & & a_{M-1} & b_{M-1} & c_{M-1} \\ & & & & & e_M & & a_M & b_M \end{pmatrix}$$

$$u = \begin{pmatrix} u_1 \\ u_2 \\ \cdot \\ u_4 \\ \cdot \\ \cdot \\ \cdot \\ u_{M-1} \\ u_M \end{pmatrix}$$

$$d = \begin{pmatrix} d_1 \\ d_2 \\ \cdot \\ d_4 \\ \cdot \\ \cdot \\ \cdot \\ d_{M-1} \\ d_M \end{pmatrix}$$

In the case of a one-dimensional system,as in Equation (4.18), $\overline{A}$ takes the following tridiagonal shape :

$$\overline{A} = \begin{pmatrix} b_1 & c_1 & & & \\ a_2 & b_2 & c_2 & & \\ & \cdot & \cdot & \cdot & \\ & & \cdot & \cdot & \cdot \\ & & & a_M & b_M \end{pmatrix}$$

The solution of these matrices requires a major consumption of computer time. Essentially, there are two types of methods that are used for the solution of the generated matrices[18,19]:

. Iterative solution

. Direct solution

The two methods are explained in more details in Appendix-B.

4.10 Mass and Energy Balances

One criterion for checking the accuracy of the finite difference approximations involves using the mass and energy balance. One form of the mass or heat balance that can be selected is the difference between the mass or energy in place at the begining and at the end of the time step, and this should be equal to the total fluids or energy production and expansion during one time step. The Mass Balance Error is therefore calculated as:

$$MBE = \frac{\Delta_t \sum_{i=1}^{m} \left\{ V_b \phi \left(\frac{s_l}{B_l} \right) \right\}_i}{q_l \Delta t} - 1 \qquad (4.21a)$$

Where:

Δ_t = the difference between the bigining and the end of time step

$= (\ldots)^{n+1} - (\ldots)^n$

f = stand for phase , oil,water or gas

m = total number of grids in the system

The Energy Balance Error is calculated as:

$$EME = \frac{EIP^{n+1} - EIP^n}{\left(q_{inj} C_{p(inj)} T_{inj} \Delta t\right) - \left(\sum_{i=1}^{m} \sum_{f=o,w,g} q_f C_{pf} T_i \Delta t\right)} \qquad (4.22a)$$

Where:

$$EIP = \sum_{i=1}^{m} V_b T \left\{ \left(\phi \sum_{f=o,w,g} \rho_f S_f C_{pf} \right) - (1-\phi) \rho_r C_{pr} \right\} \qquad (4.22b)$$

Low values for MEB and MBE are necessary conditions to satisfy before marching to the next time step.

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CHAPTER FIVE

STRUCTURE OF THE COMPUTER PROGRAMME

CHAPTER FIVE CONTENTS

FIGURES

CHAPTER FIVE
STRUCTURE OF THE COMPUTER PROGRAMME

5.1 Programme Strategy

Throughout the development of the computer model, the following objectives are borne on mind:

- programme efficiency
- storage optimization
- code flexibility
- future upgrade
- programme portability

However, complete optimization of these objectives can not be achieved in practice, because of opposing or conflicting demands.[1, 2, 3]

5.1.1 Programme Efficiency

One important consideration to increase programme efficiency is the use of a single subscribed array, instead of a multiple one (wherever possible) . However, using a single subscribed array scheme adds an additional difficulty in the coding and readability of the programme .

5.1.2 Storage Optimization

Grid ordering has a great effect on the computer work and storage requirements. Price and Coats[4] presented four different ordering systems based on optimal ordering. One of these is the standard ordering scheme, where the grids are numbered along the columns or rows , as shown in Figure 5.1 . The simulator uses the standard ordering scheme in the calculation but not in the utility subroutines. This arrangement provides efficient calculation , standard input and clear output .

i =	1	2	3	4	5
j = 1	1	6	11	16	21
2	2	7	12	17	22
3	3	8	13	18	23
4	4	9	14	19	24
5	5	10	15	20	25

FIGURE 5.1 STANDARD GRID ORDERING

5.1.3 Programme Flexibility and Future Upgrade

The structure of the programme contains a group of subroutines that can be integrated into one another (such as WELRAT and PRCOEF subroutines). These subroutines are designed to be structurally independent to minimise the future upgrade requirements and to increase programme flexibility .

5.1.4 Programme Portability

To minimise the problems of programme portability [5], we avoid the use of non-standard features of Fortran 77 and library software packages.

5.2 The Computational Scheme:

The simulator package consists of :

. main programme

. subroutine programmes

5.2.1 Main Programme

The main programme task constitutes the following :

Linkage :

The prime objective of the main programme is to link the simulator subroutines through coherent sequences of calls.

Extrapolation

The following Equation (5.1) is used to extrapolate the pressure and saturation of oil, water and gas ,as well as, temperature :

$$U^{n+1} = U^{n} + c\left(\frac{t^{n}}{t^{n+1}}\right)\left(U^{n} - U^{n-1}\right) \qquad (5.1)$$

where

U $^{n+1}$= extrapolated value of U (such as pressure, saturation or temperature)

c = extrapolated constant.

Advancing

After convergence, the pressure, saturation, temperature and material balance are advanced as follows :

$$U^{n} = U^{n+1} \qquad (5.2)$$

The flow chart in Figure 5.2 shows the execution order of these tasks

5.2.2 Subroutine Programmes

The main function of a simulator subroutine may be classified as one of the following :

A. utility Subroutine

B. calculation Subroutine

5.2.2.1 Utility Subroutines

Utility subroutines are subroutines which are not directly used in solving F.D.E's, but rather they provide the user with powerful facilities to control the input and the out of the model. These subroutines are :

. INPUT

. READ

. PRINT1

. PRINT2

. SWITCH

. INWELL

. UPDATE

Subroutine INPUT

The purpose of this subroutine is to read the simulator input data,which in its turn controls the behavior of the model. These data may be classified as :

. single-record data
. multiple-record data

Figure 4.3 shows subroutine INPUT flow chart.

Single-Record Data

This type of entries requires only one input record. Most of these data are flag type, that controls the flow of the model according to the user requests. These data are :

I. TITLE : to identify the run .

II. Number of grids
   a. NX : no of grids in x-direction
   b. NY : no of grids in y-direction

III. Programme control parameters .

a. ICHECK : check the input data for error.
b. IPRINT : index for suppressing

print-out of the initial data.

c. IPLOT : to plot the reservoir lay-out .

d. IHEAT : to set heat option on .

IV. Solution method

a. ADI: Alternating Diagonal Implicit method.

b. LSOR : Line Successive Overrelaxation method.

c. PSOR : Point Successive Overrelaxation method.

V. Fluids and rock compressibility

a. c_o : oil compressibility

b. c_w : water compressibility

c. c_g : gas compressibility

d. c_{fm} : formation compressibility

VI. Irreducible saturations

a. s_{oir} : Irreducible oil saturation

b. s_{wir} : Irreducible water saturation

c. s_{gir} : Irreducible gas saturation

VII. Reference points

a. p_{Bp} : Bubble point pressure

b. T_{rf} : Temperature reference point

VIII. Fluid densities at standard conditions.

a. osc : oil density at standard conditions

b. wsc : water density at standard conditions.

c. gsc : gas density at standard conditions.

IX. Extrapolation factor (c), see Equation (5.1).

X. Well Radius (rw)

XI. Time control parameters

a. ITIME : initial time

b. TFINAL : final time

c. DTIME : time step

d. TPRINT : time for print-out

e. MAXDT : maximum time step

f. MINDT : minimum time step

XII. Solution parameters

a. TLR : convergence tolerance

b. MAXIT : maximum number of iterations

c. w : LSOR and PSOR relaxation factor

d. NPRM : number of ADI parameter

Multiple-Record Data

Entries of this type need to be assigned to every grid in the reservoir. These entries are :

I. Reservoir geometry

a. DX : delta x

b. DY : delta y

c. TH : grid thickness

d. DEP : grid depth

II. Rock Data

a. k_x : x-direction permeability

b. k_y : y-direction permeability

c. ϕ : porosity

III. Initial Conditions

a. p_{oi} : initial oil pressure

b. s_{oi} : initial oil saturation

c. s_{gi} : initial gas saturation

d. T_i : initial temperature

Subroutine READ

Multiple-Record Data, even in a small study, usually runs into thousands of input records . Data of this type consume an immense amount of time and effort, and consequently is subject to human error. For these reasons subroutine READ is developed to ease this problem by assigning single values to all of the grids in a homogeneous reservoir or to all the grids in the region in regional reservoir . Figure 5.4 represents the subroutine flow chart.

Subroutine INWELL

The purpose of this subroutine is to read and to check the input well data . These data are :

a. well rate

b. well location

c. next well time (WELTIM)

Figure 5.8 shows the flow chart of the subroutine.

Subroutine UPDATE

This subroutine update grid identity (ID). ID defined as follows :

a. ID = 0, grid out side the reservoir boundary.

b. = 1, grid inside the reservoir boundary.

c. = 2 $\leq$ ID $\leq$ 5, shut-off well

d. =-3 $\leq$ ID $\leq$ -1, injection well

e. =-4 $\leq$ ID $\leq$ -6, production well

f. = -7, total flow rate

g. = -8, wellbore pressure

h. = -9, heat injection

Subroutine PRINT1

Prints the initial input data , if IPRINT parameter is equal to unity. The purpose of this print-out is to check the validity of the input data . The flow chart can be seen in Figure 5.9 .

Subroutine PRINT2

The function of PRINT2 is to print the simulator results after each PTIME. See Figure 5.22 .

Subroutine SWITCH

Switches from one grids ordering scheme to anther, according to the simulator mode . flow chart of Figure 5.10 demonstrates the subroutine process.

5.2.2.2 Calculation Subroutines

These subroutines are directly involve in solving F.D.E :

. TIMCON
. START
. TIMCAL
. PRCOEF
. WELRAT
. NEWSAT
. HTCOEF
. MB
. ADI
. LSOR

. PSOR

. THOMAS

. prop

Subroutine TIMCON

TIMCON subroutine calculates all independent time variables, for example the geometric constant part of the transmissibility in the fluid flow using Equation (4.4b) for all the grid in the system, as well as the conduction constant in the energy Equation. This subroutine also calculates grid bulk volume as follows:

$$V_{bi} = \Delta x_i \Delta y_i \Delta h_i \qquad (5.3)$$

Where 'i' represent grid point i.

The subroutine flow chart is given in Figure 5.5.

Subroutine START

The function of this subroutine is to calculate the following :

a. Initial saturation and pressure of water from Equations (4.1) and (4.2a).

b. Initial pressure of gas using Equation (4.2b).

c. Initial fluids (oil , water and gas) and energy in place as follows :

$$FLUTI = \frac{V_b \phi S_f}{B_f} \qquad (5.4)$$

Where FLUTI represents initial fluid in place (fluid = oil , water or gas).

d. Initial heat in place as :

$$HTTI = V_b\left\{\phi\left(C_{po}\rho_o S_o + C_{pw}\rho_w S_w + C_{pg}\rho_g S_g\right) + (1-\phi)\rho_r C_{pr}\right\} \qquad (5.5)$$

Where HTTI represents initial energy in place

Figure 5.6 shows the subroutine flow chart .

Subroutine TIMCAL

If time-step is not specified in subroutine Input, then this subroutine is used to calculate the maximum Time-step. The calculation is performed by applying Equations (4.19). The flow of this subroutine is illustrated in Figure 5.7.

Subroutine PRCOEF

The purpose of this subroutine is to calculate the pressure coefficient matrix by using Equations (4.7) . See Figure 5.11.

Subroutine WELRAT

A well in the subroutine is identified (ID) by its flow rate. WELRAT subroutine utilises this (ID) to calculate oil , water and gas rate as by using Equations (4.12), (4.13), (4.14) and (4.15) as shown in Figure 5.12. The following ID index is used to identify wells type:

I. Constant injection rate :

ID = -1 for oil

ID = -2 for water

ID = -3 for gas

II. Constant production rate :

ID = -4 for oil

ID = -5 for water

ID = -6 for gas

III. Constant total rate

ID = -7

IV. Constant wellbore pressure

ID = -8

V. Heat injection

ID = -9

Subroutine NEWSAT

NEWSAT calculates explicitly the new oil, water and gas saturations using Equations (4.9), and updates the water and gas pressure using Equations (4.2) . Figure 5.13 shows The subroutine flow chart.

Subroutine HTCOEF

The heat coefficient matrix is calculated using Equations(4.10). The flow chart is given in Figure 5.14.

Subroutine ADI

Subruotine ADI is one of three matrix solver subroutines . ADI solves the generated coefficient matrix from PRCOEF or HTCOEF subroutines by the following iterative procedure. Equations (4.8) and (4.11) are divided into two steps :

a. x-direction sweep

b. y-direction sweep

a. x-direction sweep : at this step we are only solving for W,M and E values, so Equations (4.8) and (4.11) are written as follows:

$$a_M \dot{U}_W + b_{xM} \dot{U}_M + c_M \dot{U}_E = d_{xM} \qquad (5.6a)$$

Thomas algorithm is used to solve the resulted tridiagonal matrix from Equation (5.6a) .

b . y-direction sweep : at this step , the $\dot{U}s$ are used in the solution of N, M and S values. The pressure and energy Equations (4.8) and (4.11) respectively , are arranged as follows :

$$e_M U_N^{k+1} + b_{yM} U_M^{k+1} + f_M U_S^{k+1} = d_{yM} \qquad (5.6b)$$

New solution , of pressure or temperature ,is obtained from solving Equation (5.6b). This solution then will be used in the following equation to calculate the r value, which will be tested against a given tolerance for convergence

$$r = p^{k+1} - p^k \qquad (5.6c)$$

In the ADI flow chart Figure 5.17, 'X' is symbolizing pressure or temperature .

Subroutine PSOR

This method arranges Equations (4.8) and (4.11) to account only for one grid. As shown in figure 5.18, iteration process is conducted over the following equations:

$$O_M = \frac{1}{b_M}\left(d_M - e_M U_N^{K+1} - a_M U_W^{K+1} - c_M U_E^{K} - f_M U_S^{K}\right) \qquad (5.16a)$$

Then O relaxed by the following equation:

$$U_M^{K+1} = U_M^{k} - \omega\left(O_M - U_M^{K}\right) \qquad (5.16b)$$

Where

k = iteration level

ω = relaxation parameter

This iterative process is continue until the constrains given by Input subroutine are satisfied.

Subroutine LSOR

The LSOR method reduces the two-dimensional problem of Equations (4.8) and (4.11) to the following one-dimensional problem : (see Figure 5.19)

$$a_M O_W + b_M O_M + c_M O_E + = \ddot{d}_M \qquad (5.17a)$$

Where:

$$\ddot{d}_M = d_M - e_M U_M^{k+1} - f_M U_M U_E^{k} \qquad (5.17b)$$

Equation (5.17a) generates a tridiagonal matrix which can be solved by Thomas algorithm. The value of O can be relaxed by the following equation:

$$U_M^{k+1} = U^k - \omega\left(O_M - U_M^K\right) \qquad (5.17c)$$

Subroutine THOMAS

ADI and LSOR methods generate , through there solving process , a tridiagonal matrix of the following form.

$$\overline{A} = \begin{pmatrix} b_1 & c_1 & & & \\ a_2 & b_2 & c_2 & & \\ & \cdot & \cdot & \cdot & \\ & & \cdot & \cdot & \cdot \\ & & & a_M & b_M \end{pmatrix}$$

The solution is divided into two steps. The first step is the following

$$W_i = b_i - \frac{a_i c_{i-1}}{W_{i-1}} \text{ with } \quad W_1 = b_1 \qquad (5.18a)$$

$$Q_i = d_i - \frac{a_i Q_{i-1}}{W_i} \text{ with } \quad Q_1 = \frac{d_1}{b_1} \qquad (5.18b)$$

The second step is Back substitution ...

$$u_M = Q_M \qquad (5.18c)$$

$$u_i = Q_i - \frac{c_i u_{i+1}}{W_i} \qquad (5.18d)$$

These steps are illustrated in the subroutine flow chart, Figure 5.20

Subroutine PROP

PROP subroutine function is to provide the calculation subroutines with the following fluid and rock properties :

I. Rock

a. porosity, ϕ = f(p)

b. relative permeability , kr = f(s)

c. capillary pressure, pc = f(s)

d. thermal conductivity, kh = f(T)

e. specific heat, cp = f(T)

II. Fluid

a. formation volume factor, B = f(p,T)

b. density, ρ = f(p,T)

c. viscosity, m = f(p,T)

d. solution gas, Rs = f(p,t)

e. thermal conductivity, kh = f(T)

f. specific heat, cp = f(T)

The properties can be entered as look up tables, or correlations. The flow chart is shown in Figure 5.15.

Subroutine lookup

This subroutine uses Sequential search and linear interpolation between tables values. The efficiency of the subroutine is greatly improved if the entries are equally spaced[6]. Figure 5.16 shows the flow chart of the subroutine.

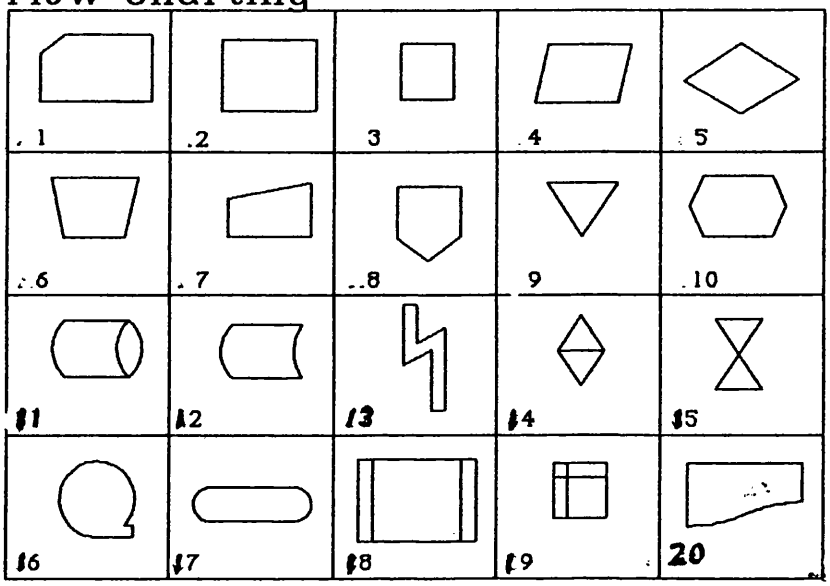
Subroutine MB

Subroutine MB is executed at the end of each time step. The function of subroutine MB is to calculate fluid mass balance and heat balance errors. Equations (4.21) and (4.22) are used in the calculation. Figure 5.21 shows the subroutine flow chart.

5.3 Flow Charts

The simulation model consists of a complex collection subroutines. The subroutines are tied together by the main or control programme. the flow chart symbols and the subroutines flow charts are listed below.

Flow Charting



CODE	SYMBOLS	CODE	SYMBOLS
1	PUNCH CARD	11	TERMINAL/INTERUPT
2	PROCESS	12	INLINE STORE
3	CORE\AUX OPERATION	13	COMMUNICATION
4	INPUT\OUTPUT	14	BOTH-AND
5	DECISION	15	EITHER-OR
6	MANUAL OPERATIONS	16	TAPE OPERATION
7	MANUAL INPUT	17	START/END
8	OFF PAGE	18	P. TAPE
9	MERGE	19	PREPARATION
10	DO-LOOP/CONTINUE	20	DOCUMENT

MAIN PROGRAMME FLOW CHART

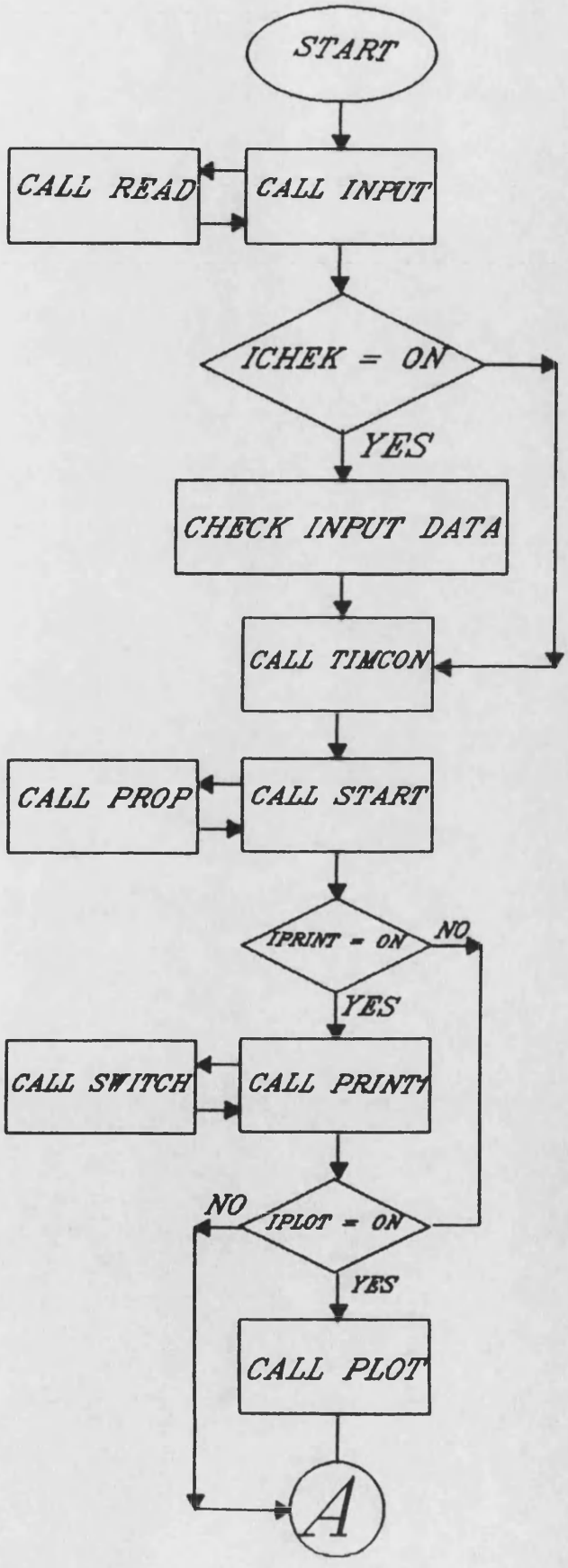
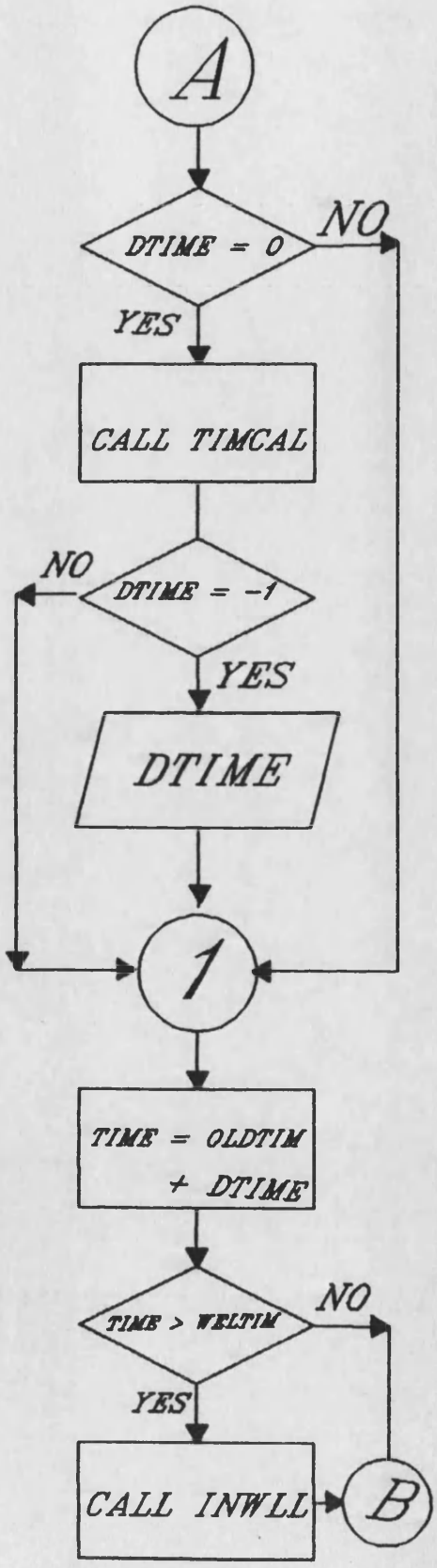


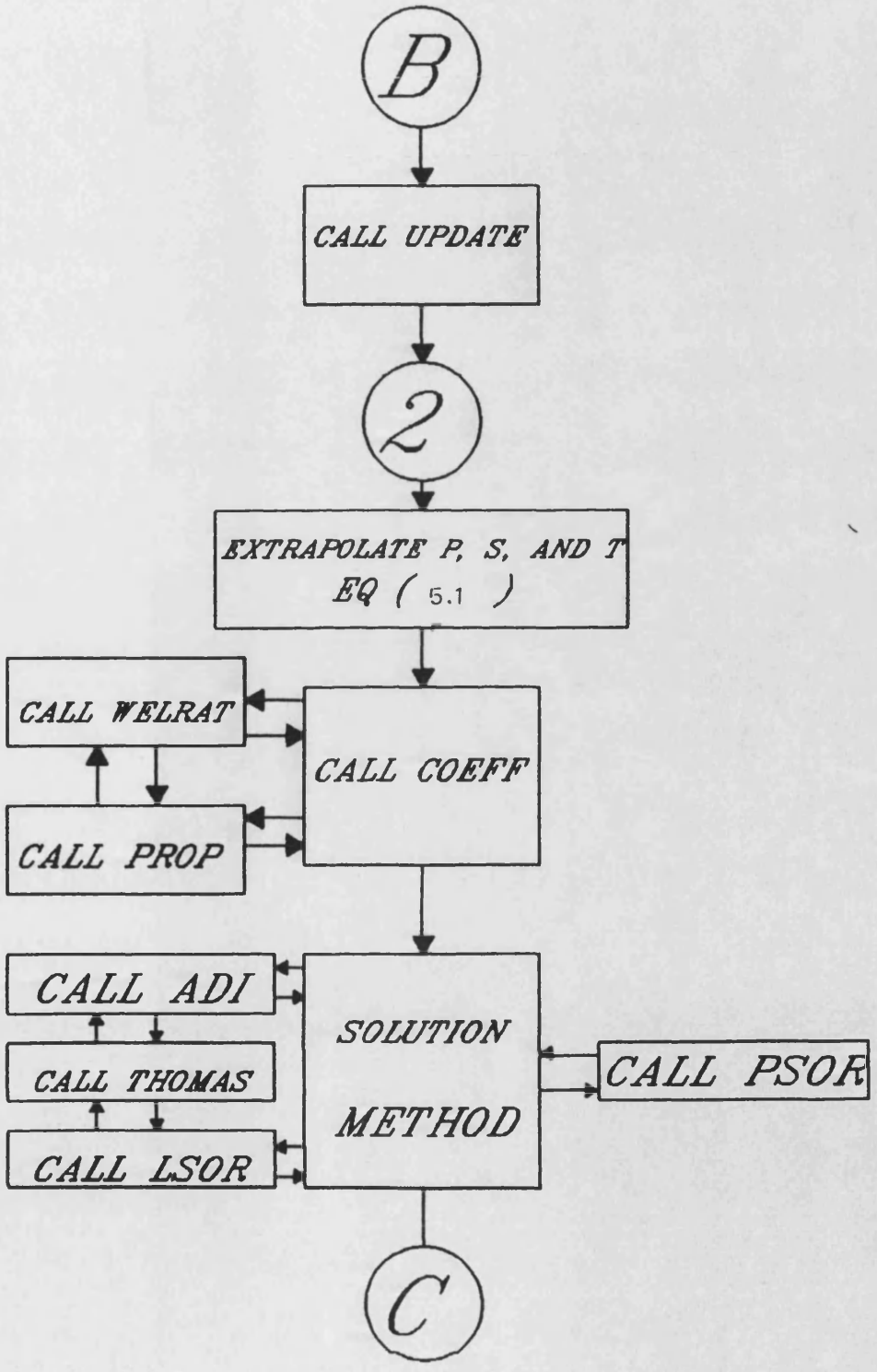
FIGURE 5.2 MAIN PROGRAMME FLOW CHART

Cont. MAIN PROGRAMME FLOW CHART



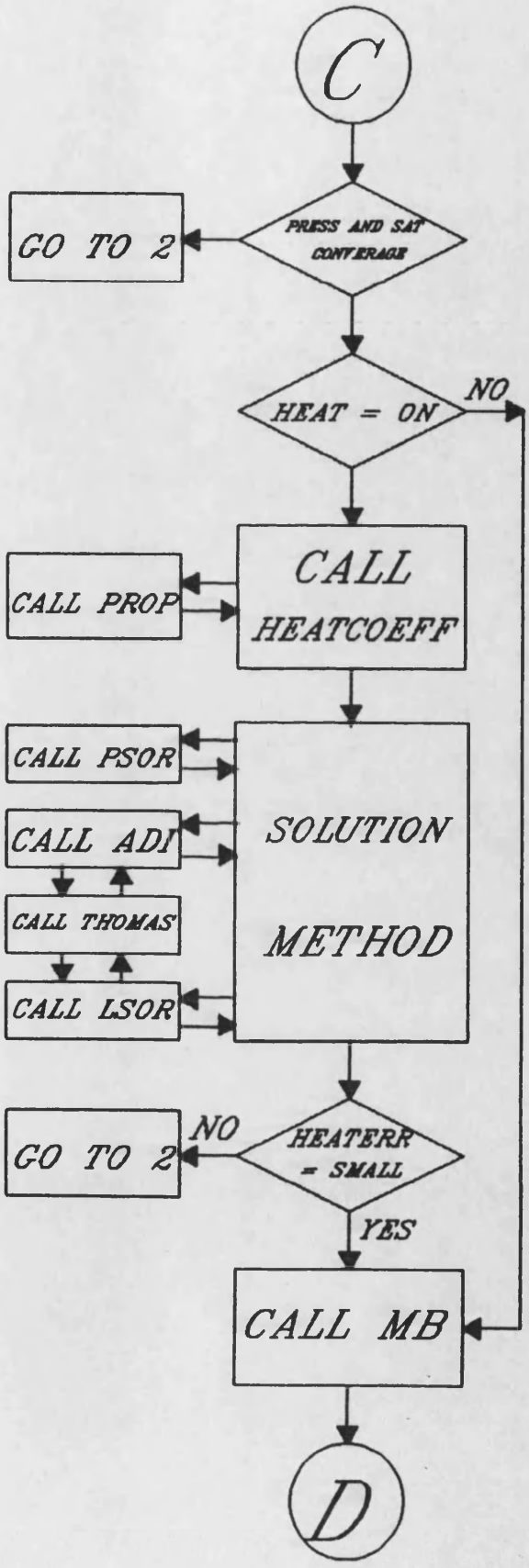
cont. FIGURE 5.2 MAIN PROGRAMME FLOW CHART

Cont. MAIN PROGRAMME FLOW CHART



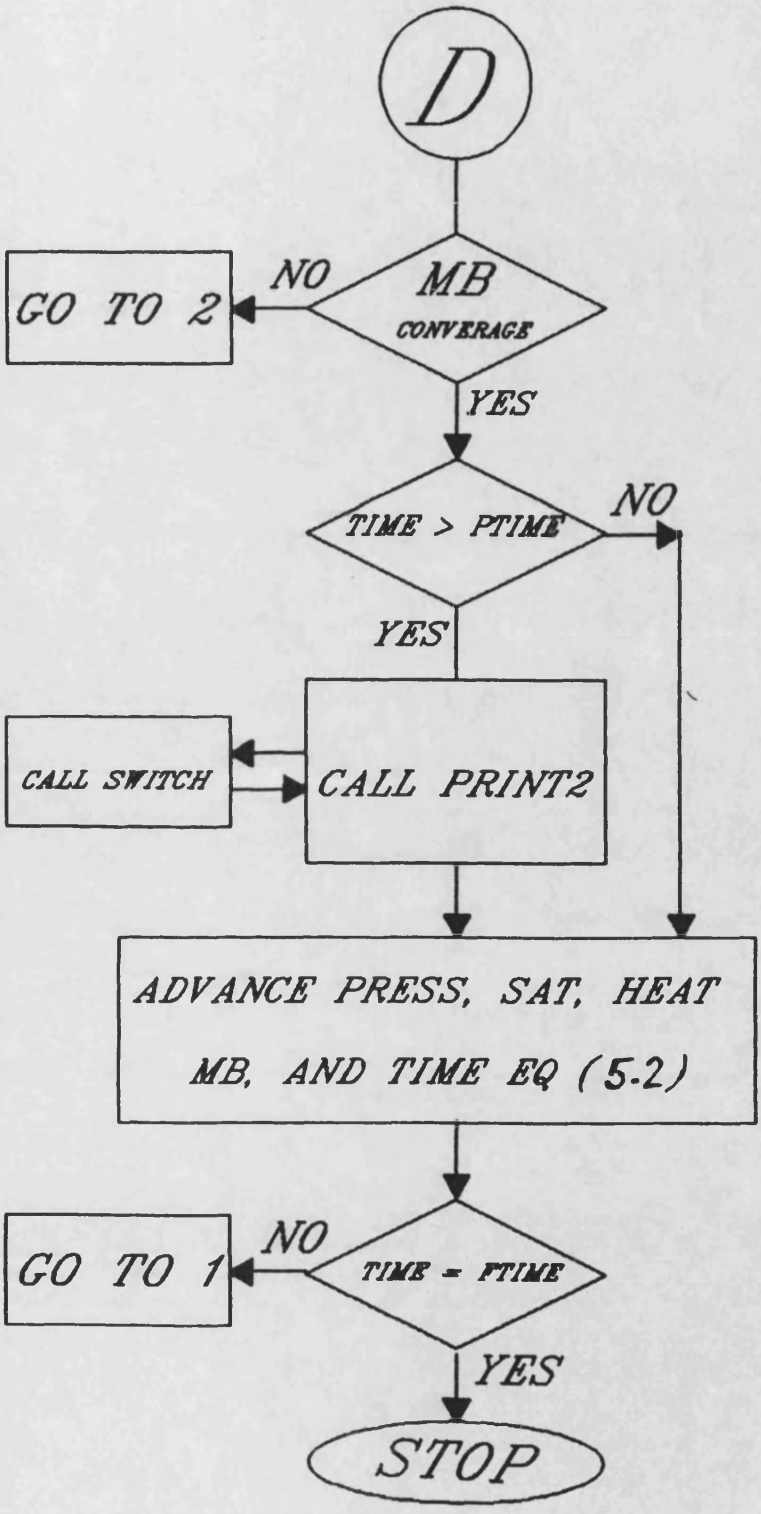
cont. FIGURE 5.2 MAIN PROGRAMME FLOW CHART

Cont. MAIN PROGRAMME FLOW CHART



cont. FIGURE 5.2 MAIN PROGRAMME FLOW CHART

Cont. MAIN PROGRAMME FLOW CHART



cont. FIGURE 5.2 MAIN PROGRAMME FLOW CHART

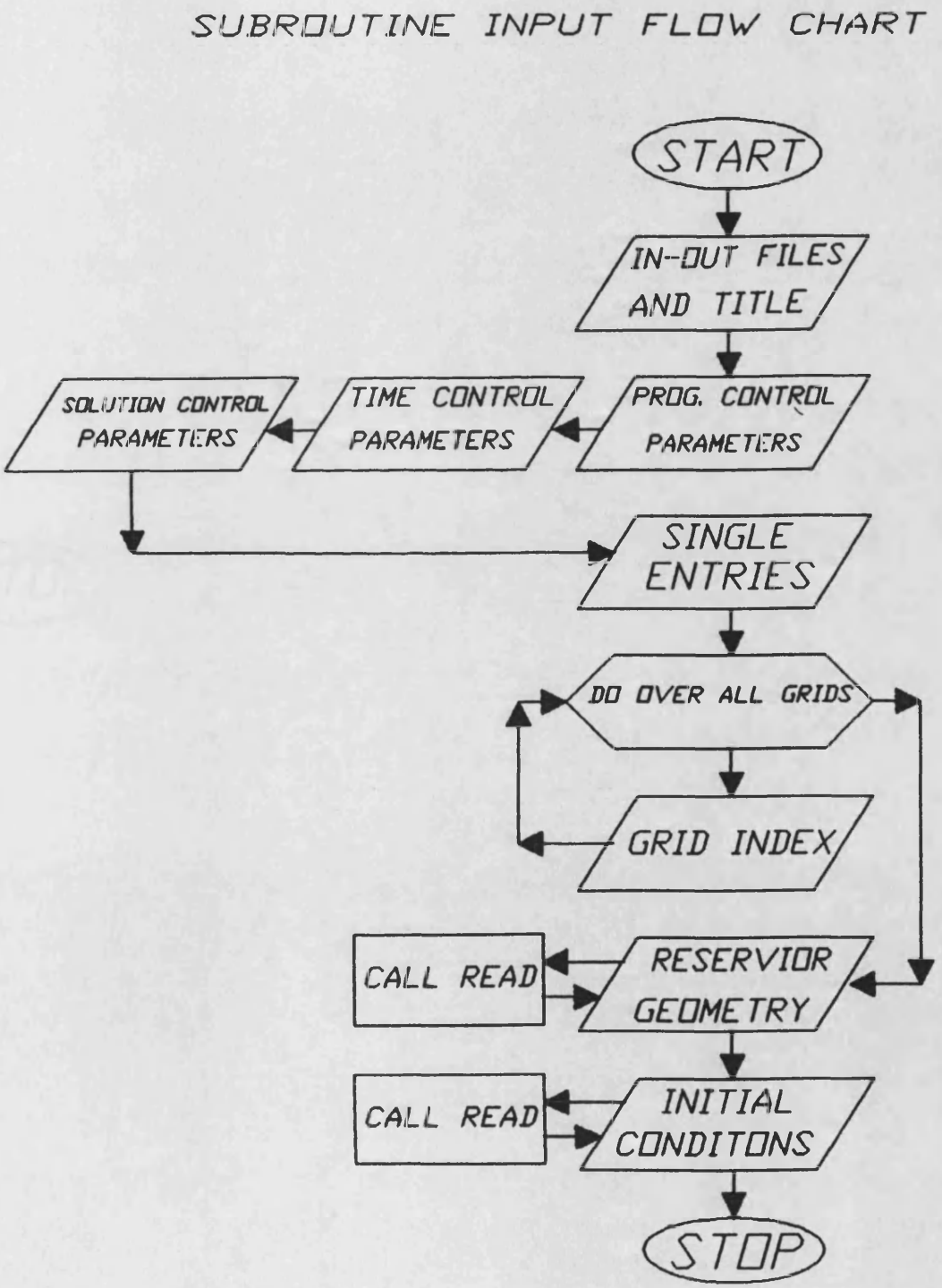


FIGURE 5.3 SUBROUTINE INPUT FLOW CHART

SUBROUTINE READ FLOW CHART

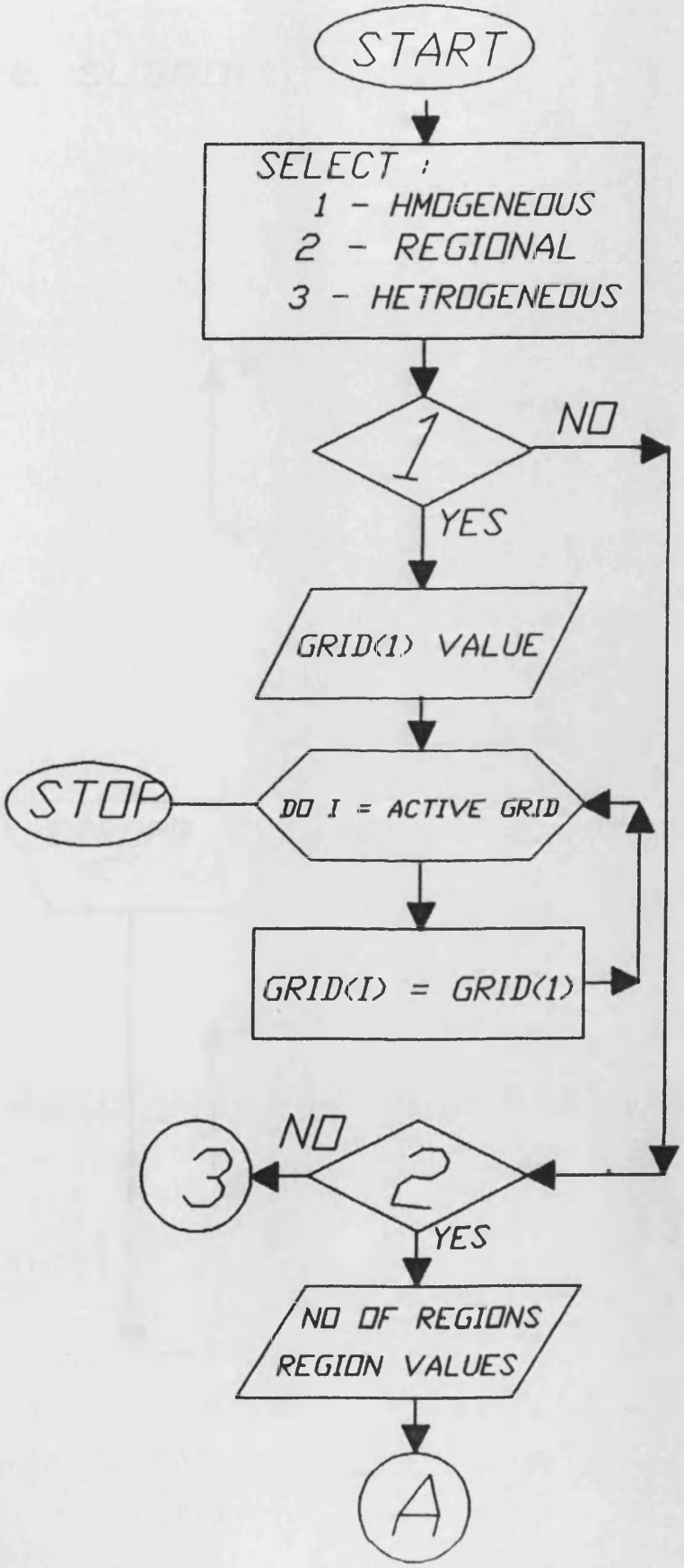
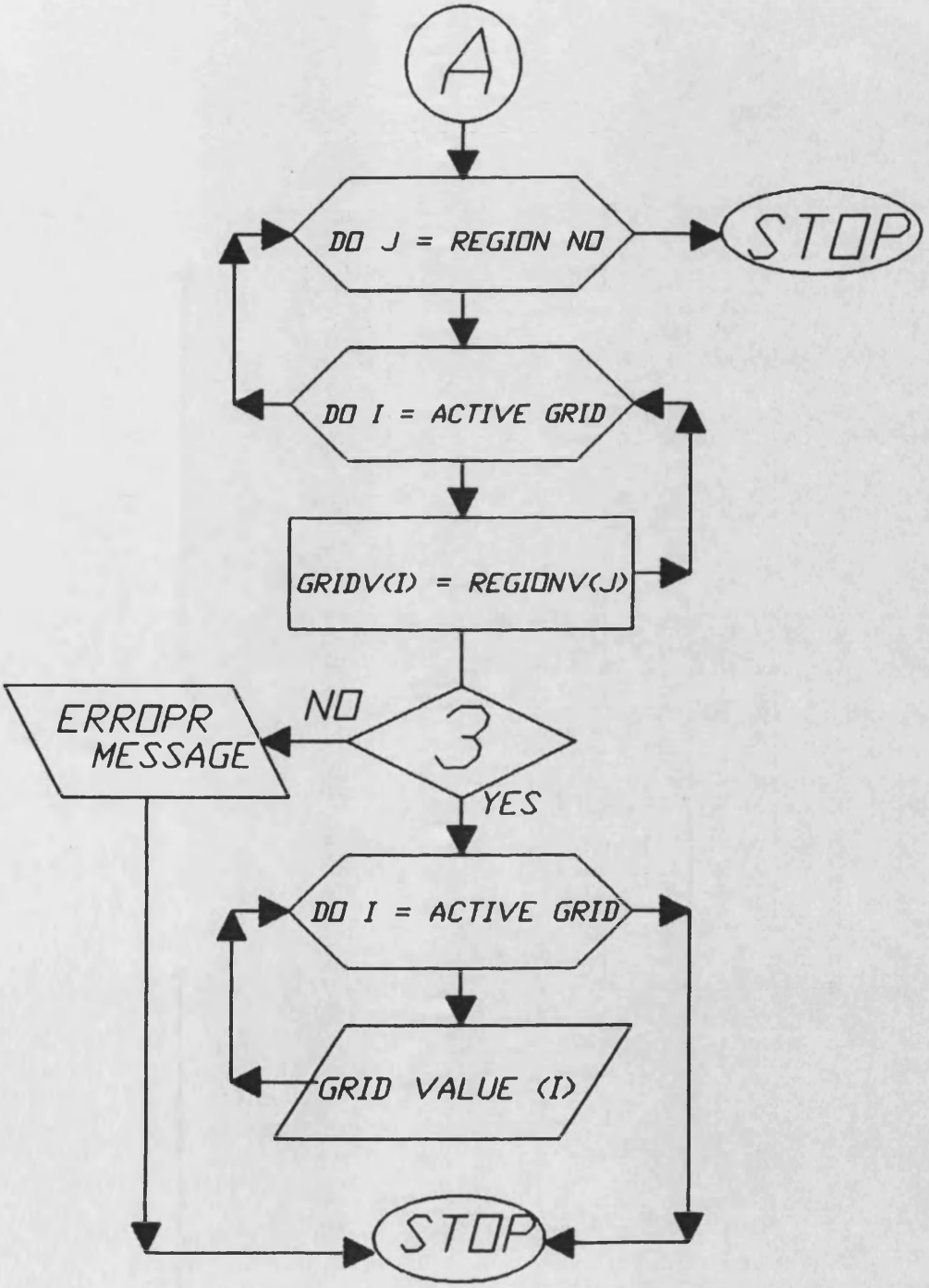


FIGURE 5.4 SUBROUTINE READ FLOW CHART

Cont. SUBROUTINE READ FLOW CHART



cont. FIGURE 5.4 SUBROUTINE READ FLOW CHART

SUBROUTINE TIMCON FLOW CHART

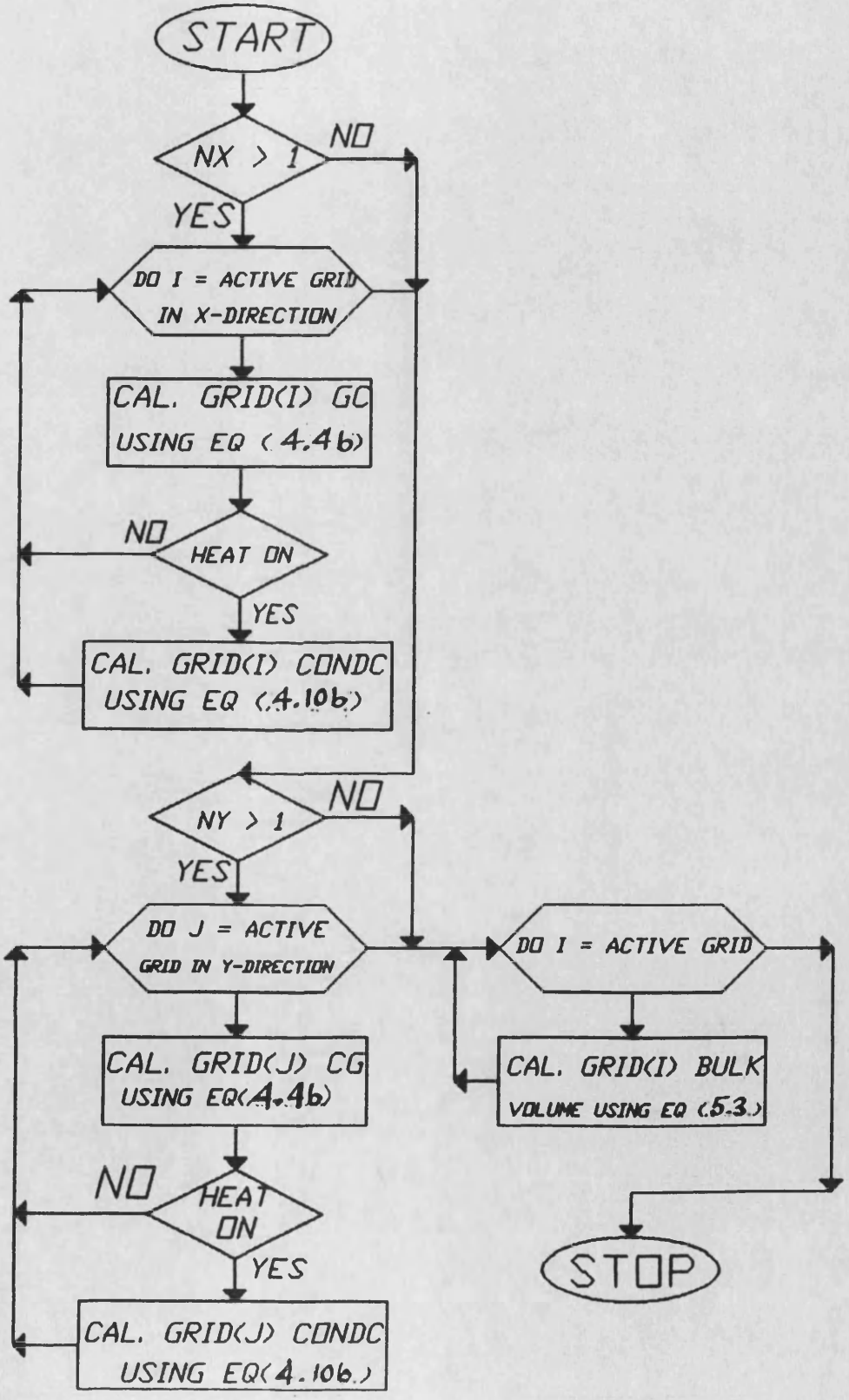


FIGURE 5.5 SUBROUTINE TIMCON FLOW CHART

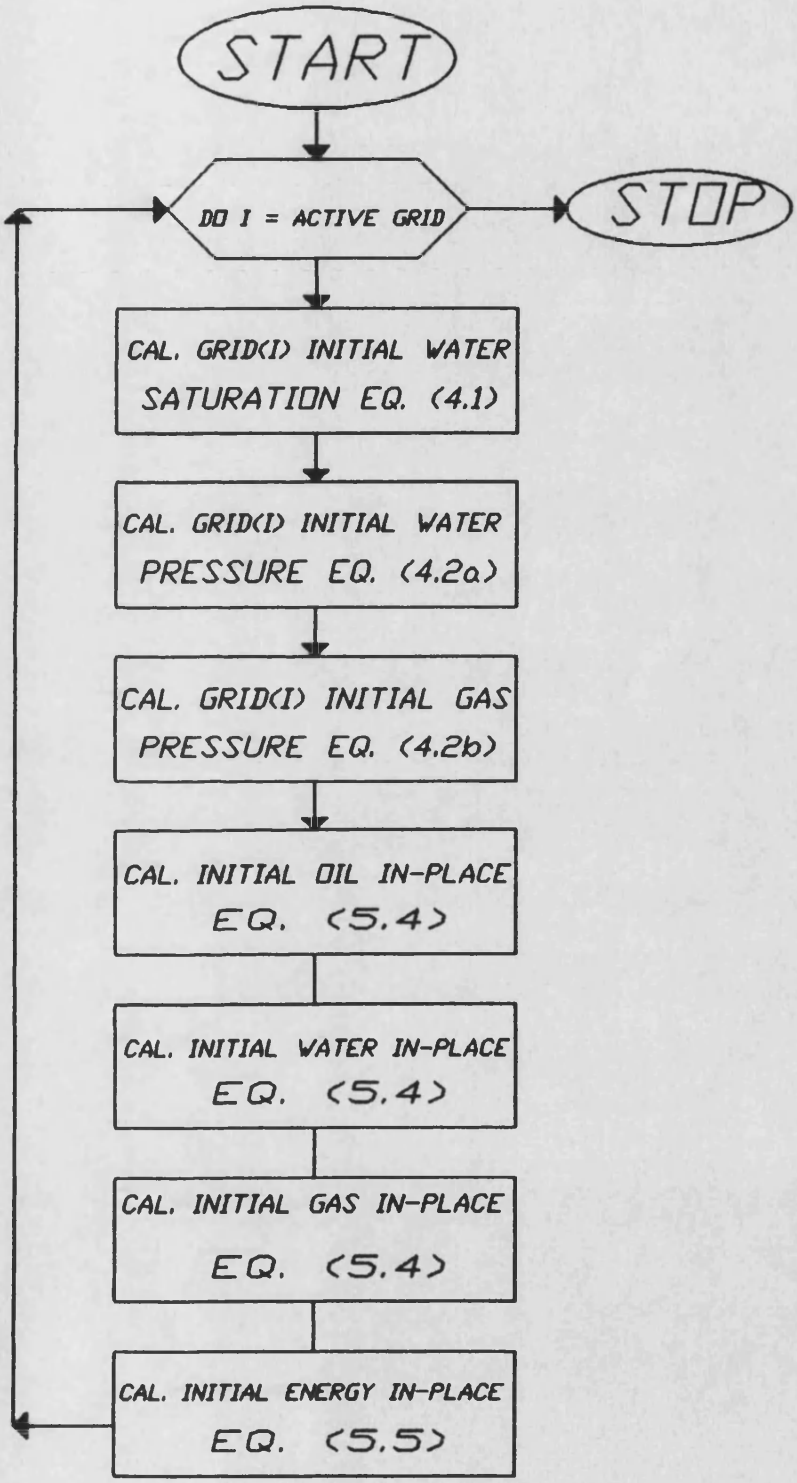


FIGURE 5.6 SUBROUTINE START FLOW CHART

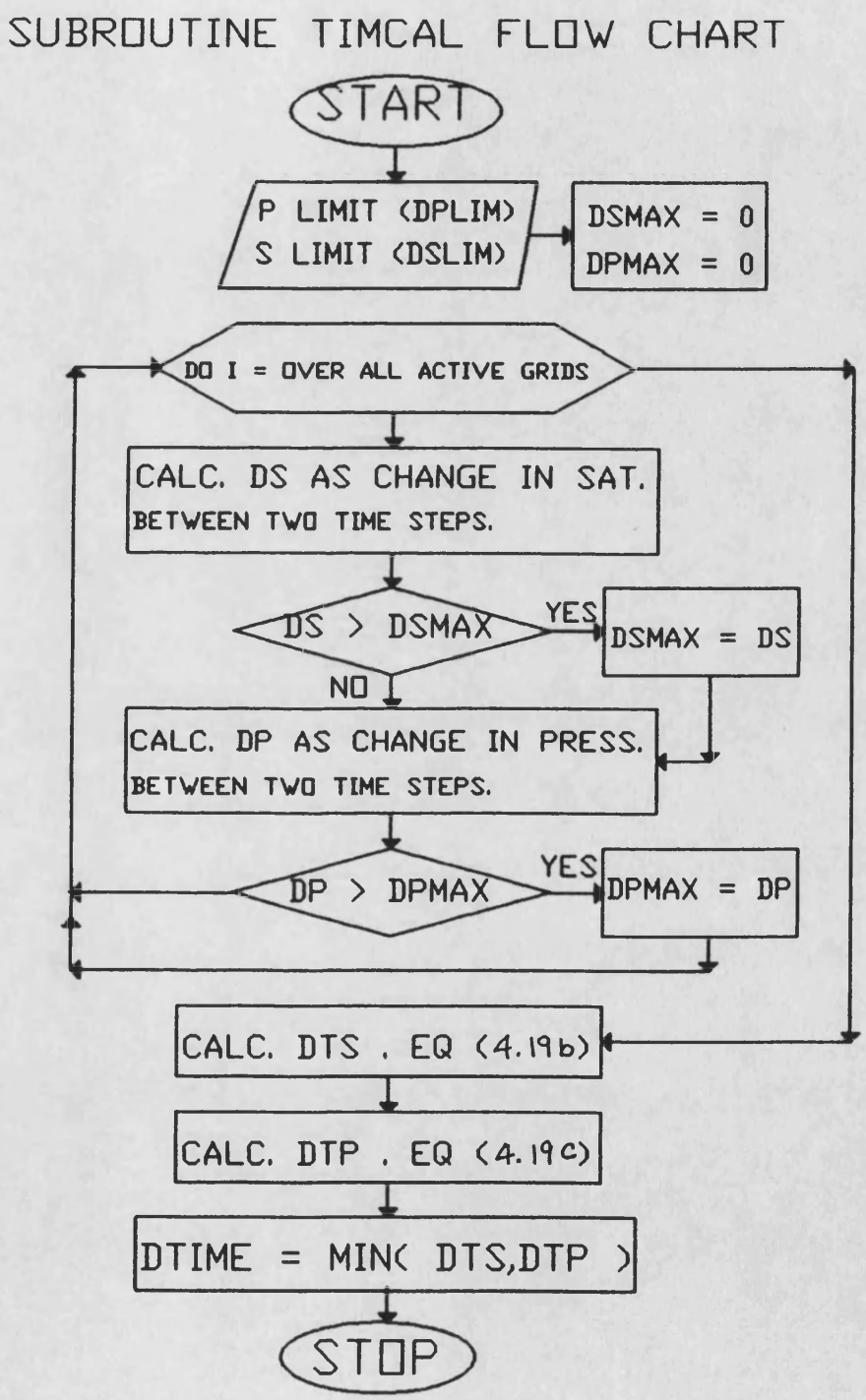


FIGURE 5.7 SUBROUTINE TIMCAL FLOW CHART

SUBROUTINE INWELL FLOW CHART

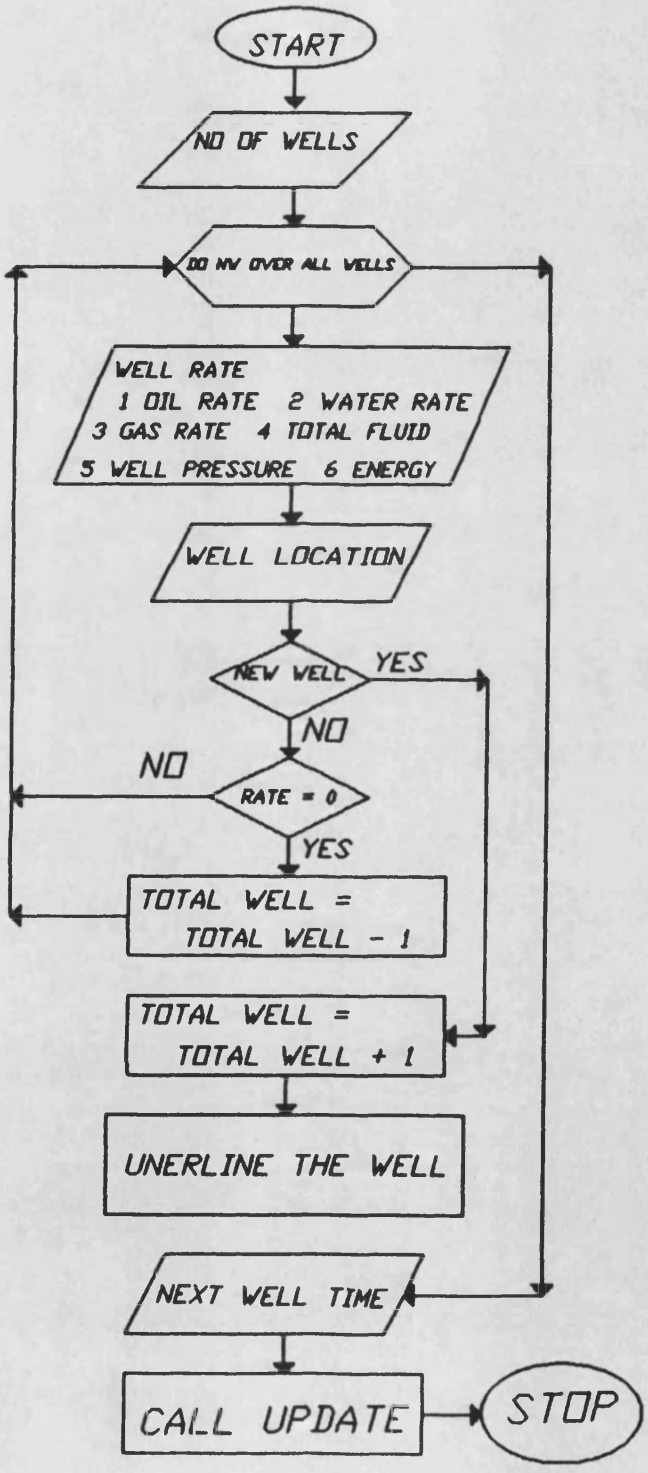


FIGURE 5.8 SUBROUTINE INWELL FLOW CHART

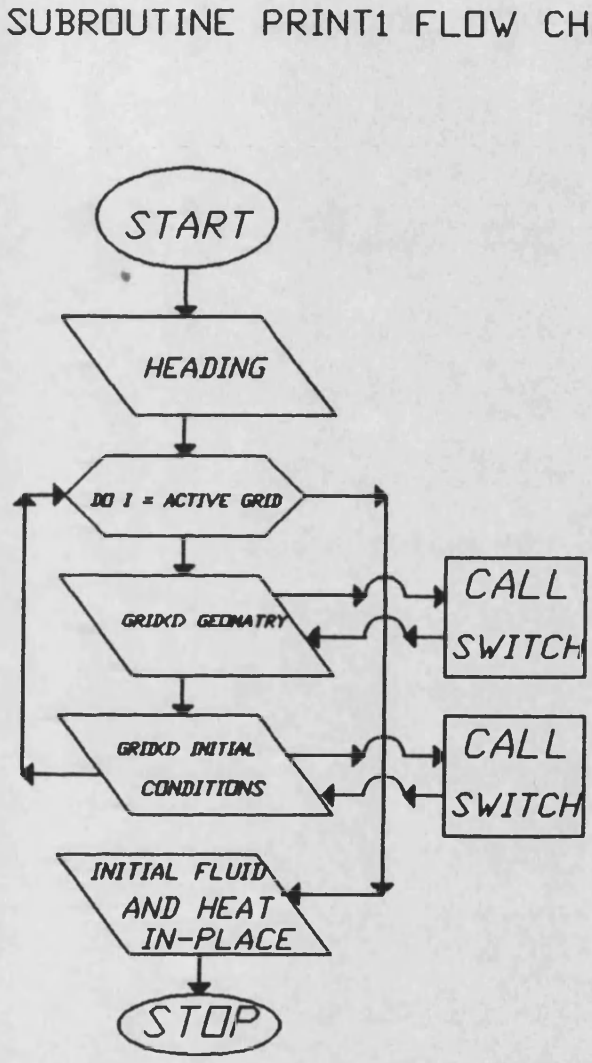


FIGURE 5.9 SUBROUTINE PRINT1 FLOW CHART

SUBROUTINE SWITCH FLOW CHART

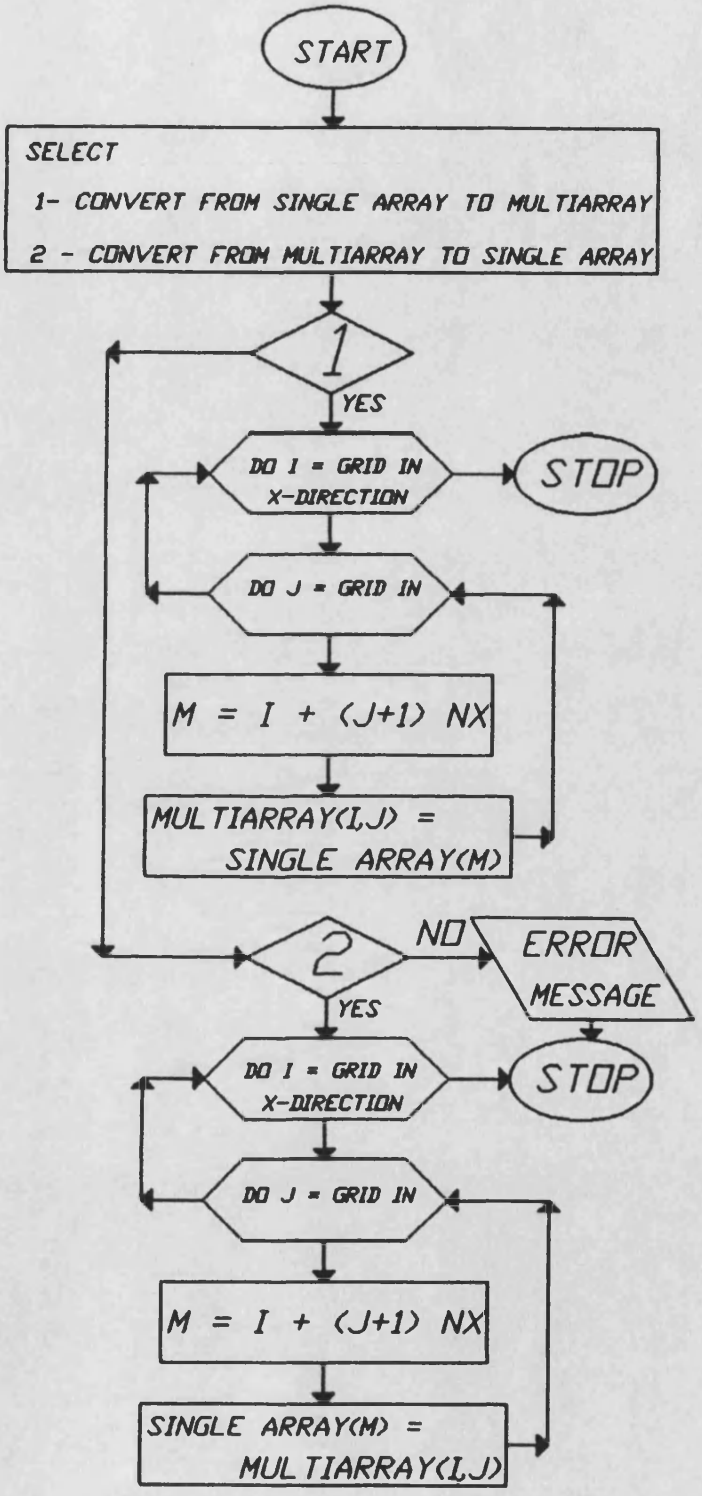


FIGURE 5.10 SUBROUTINE SWITCH FLOW CHART

SUBROUTINE PRCOEF FLOW CHART

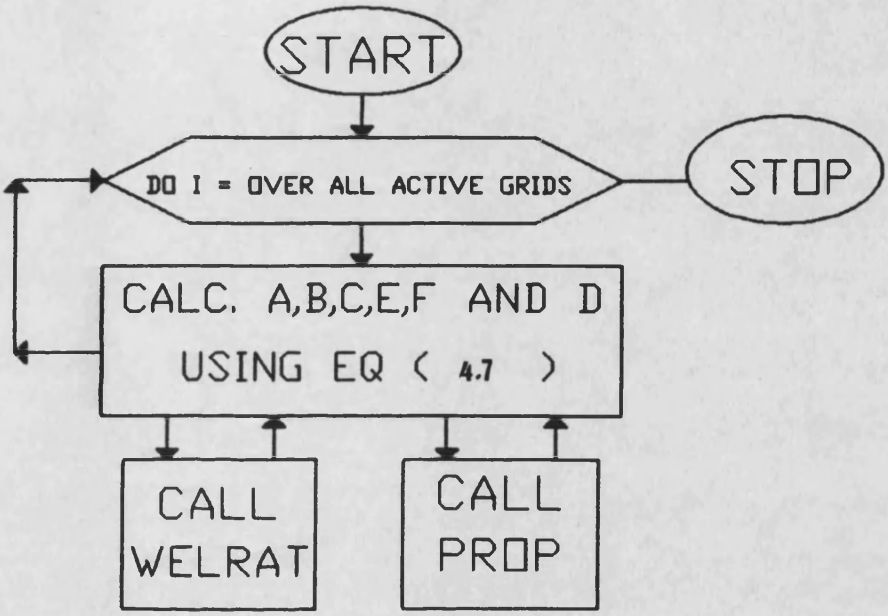


FIGURE 5.11 SUBROUTINE PRCOEF FLOW CHART

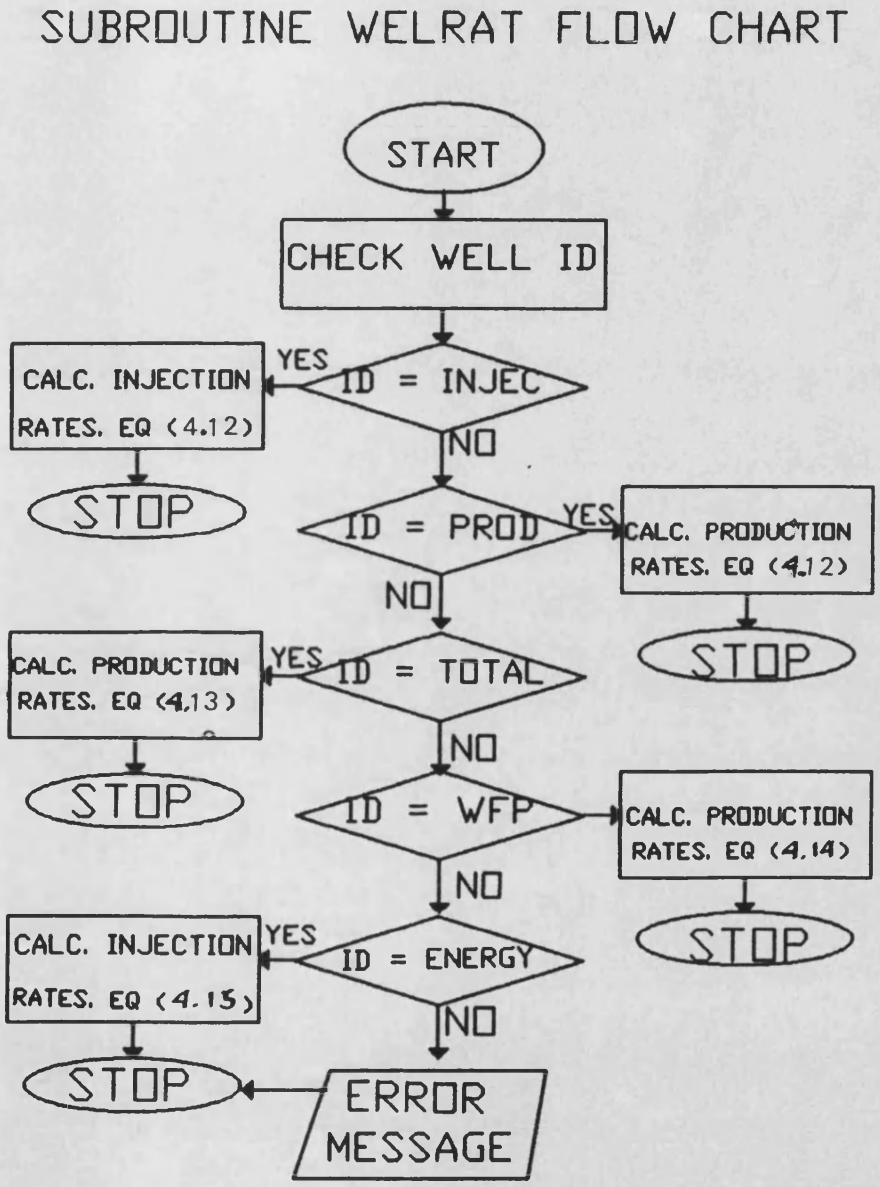


FIGURE 5.12 SUBROUTINE WELRAT FLOW CHART

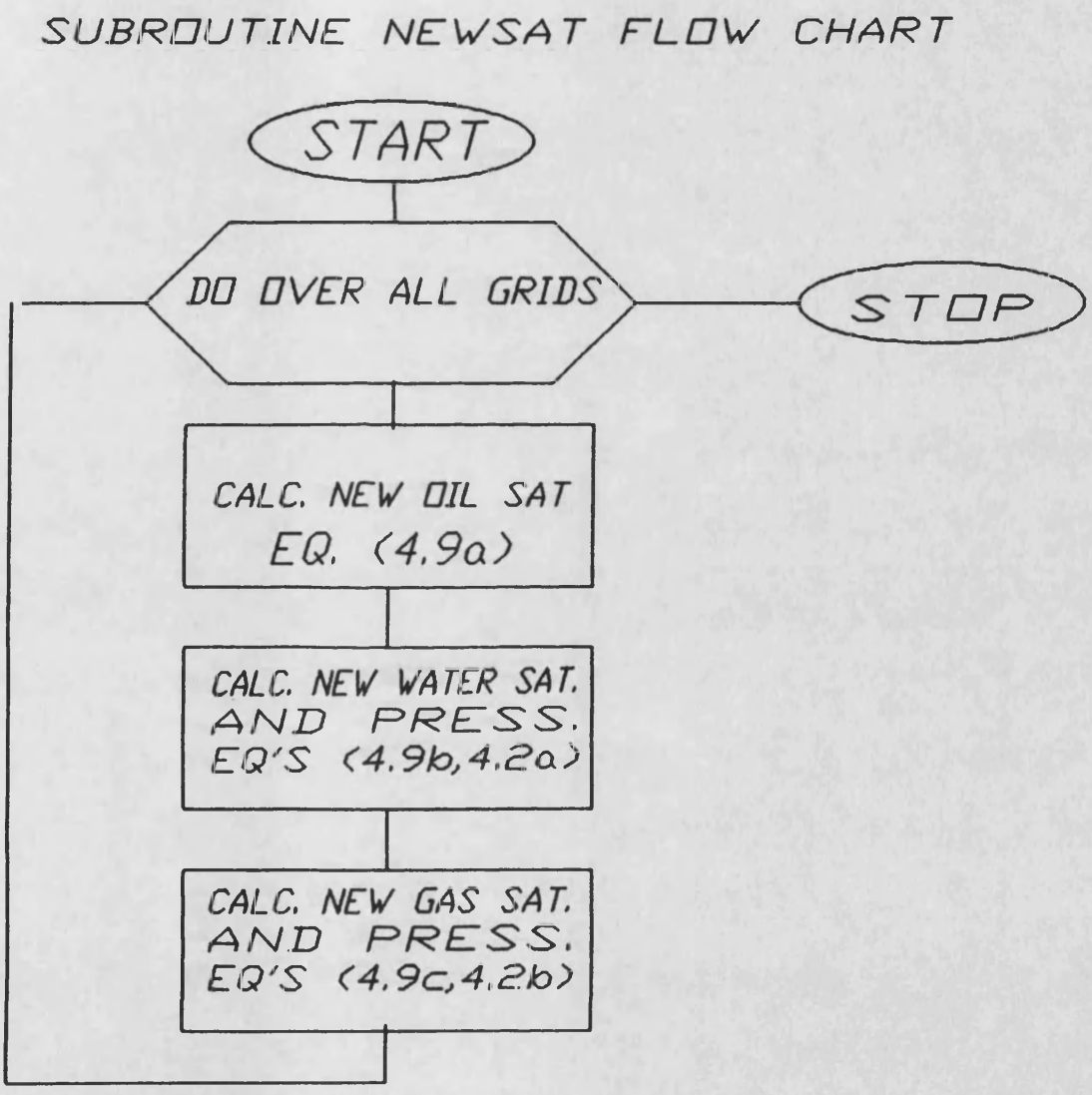


FIGURE 5.13 SUBROUTINE NEWSAT FLOW CHART

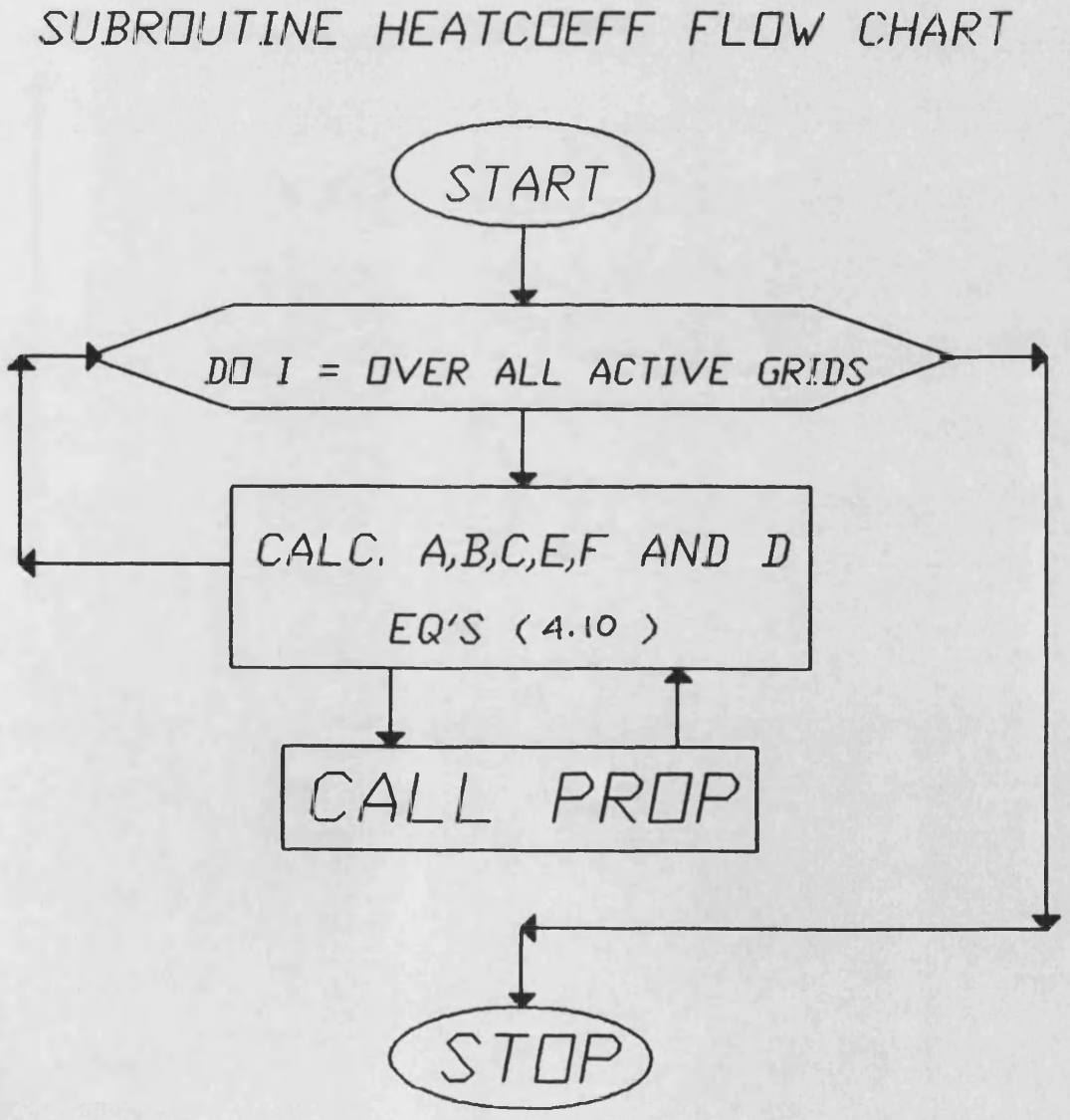


FIGURE 5.14 SUBROUTINE HTCOEF FLOW CHART

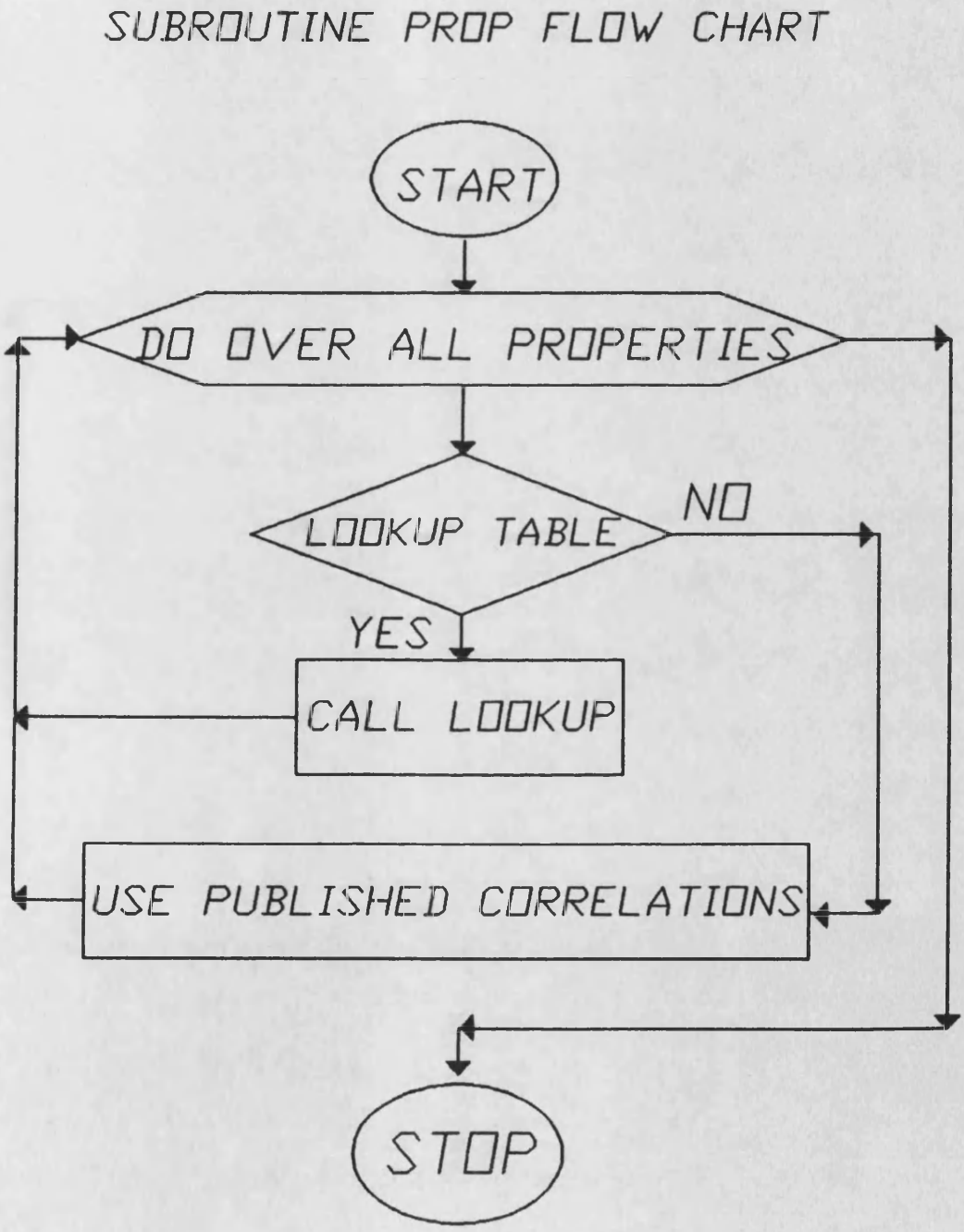


FIGURE 5.15 SUBROUTINE PROP FLOW CHART

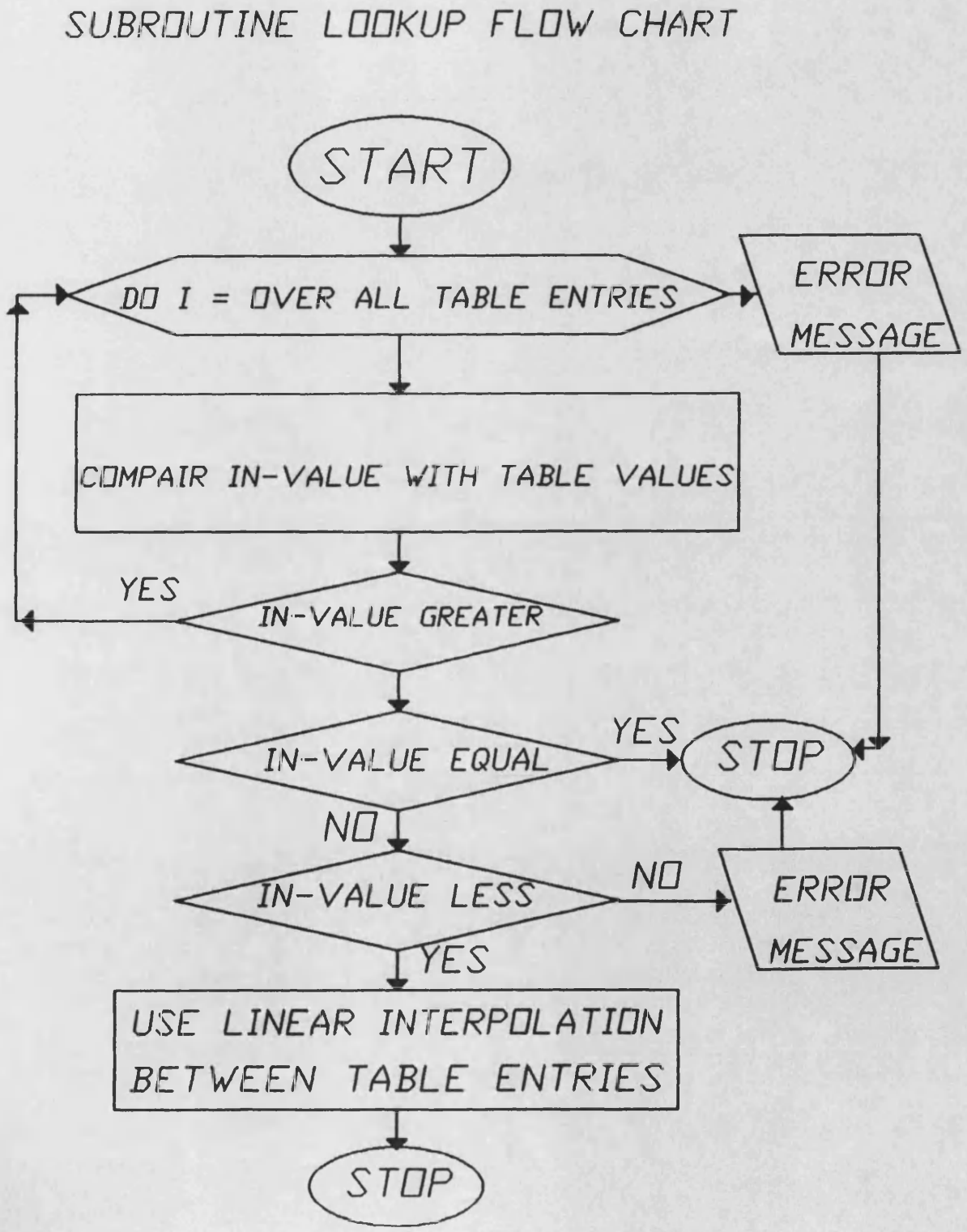


FIGURE 5.16 SUBROUTINE LOOKUP FLOW CHART

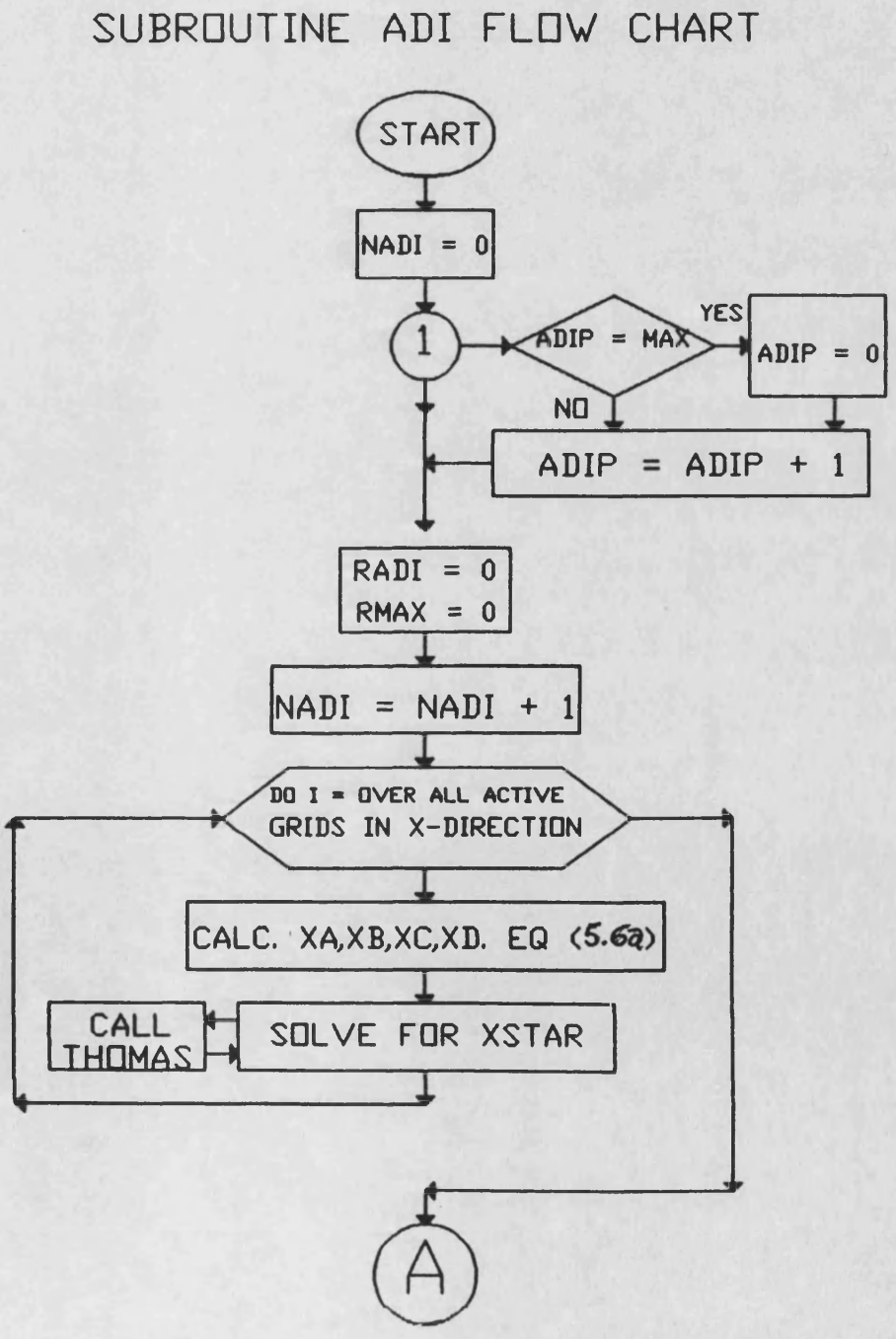
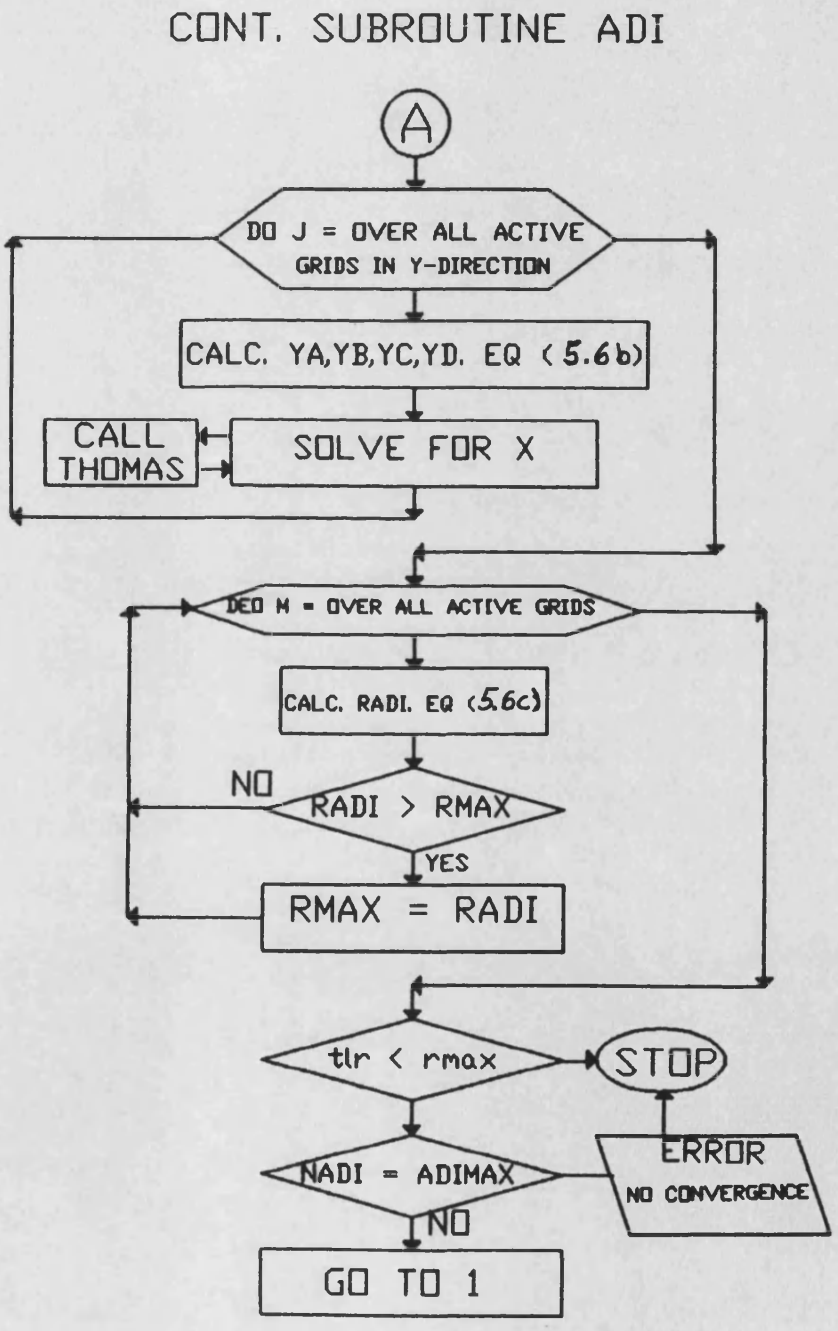


FIGURE 5.17 SUBROUTINE ADI FLOW CHART



cont. FIGURE 5.17 SUBROUTINE ADI FLOW CHART

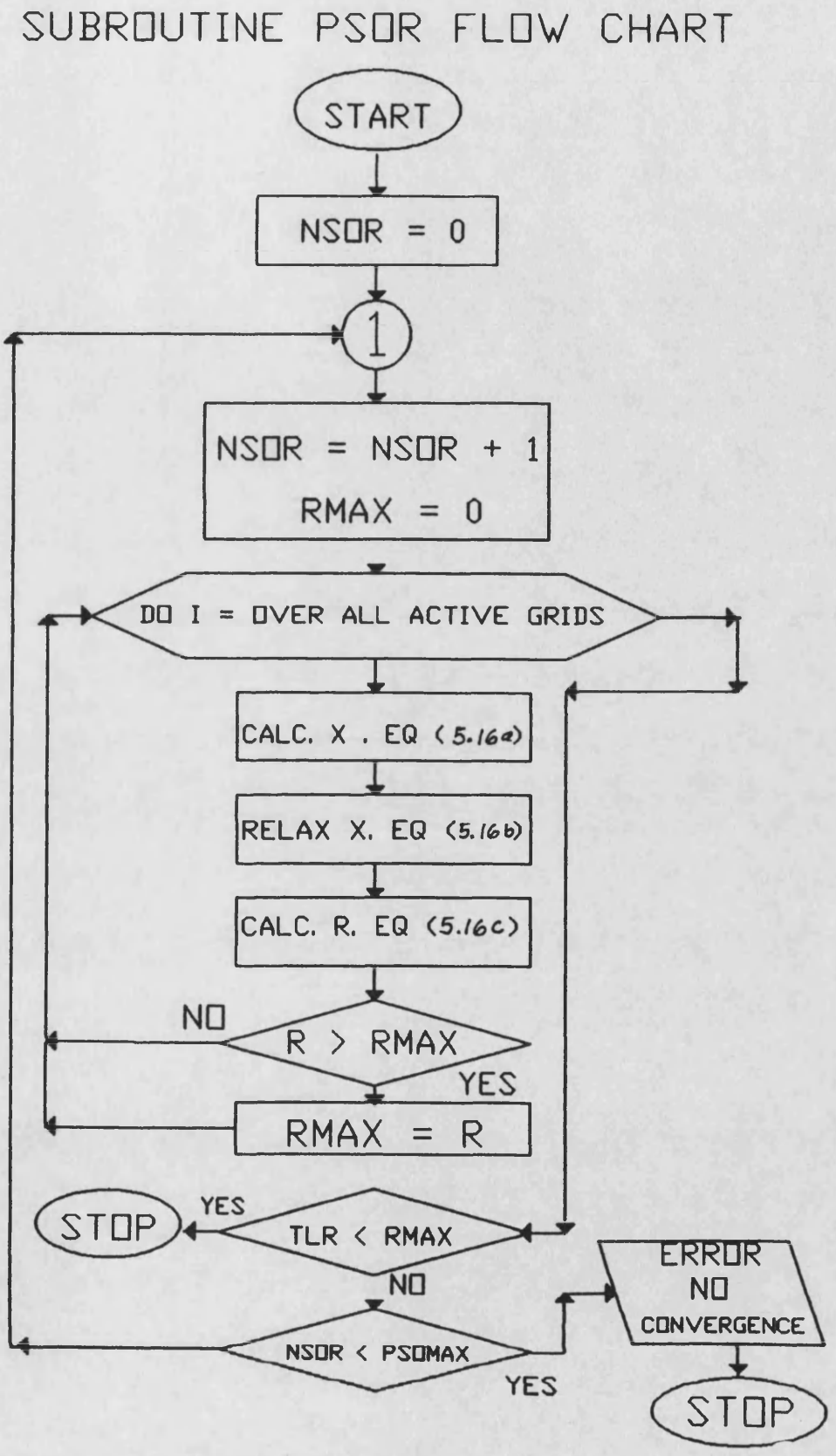


FIGURE 5.18 SUBROUTINE PSOR FLOW CHART

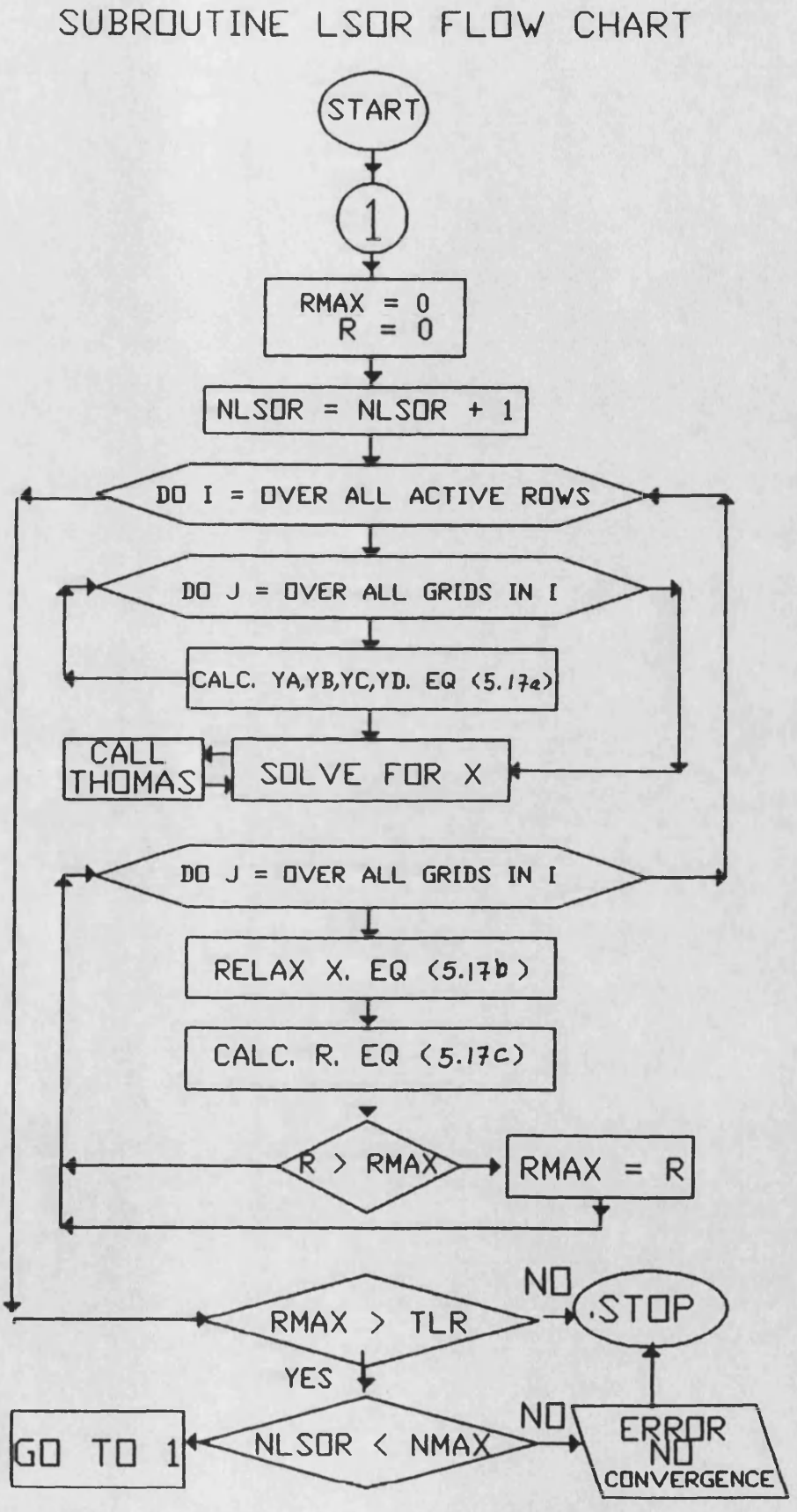


FIGURE 5.19 SUBROUTINE LSOR FLOW CHART

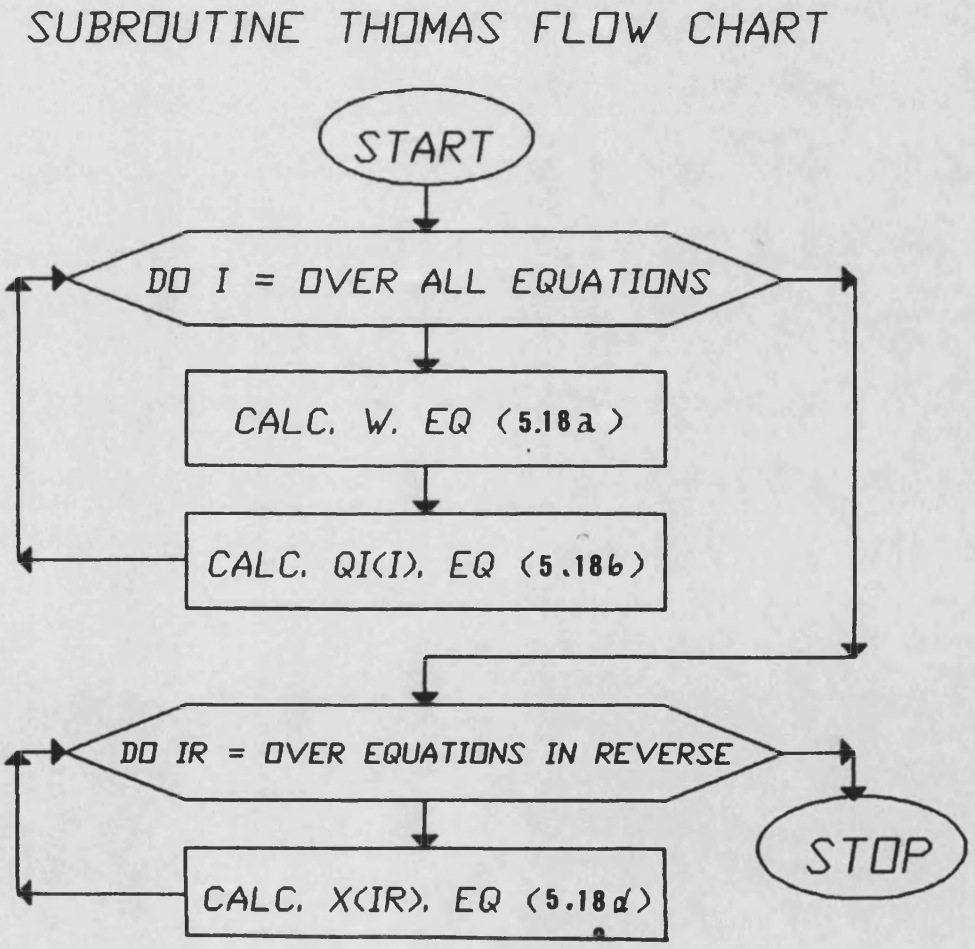


FIGURE 5.20 SUBROUTINE THOMAS FLOW CHART

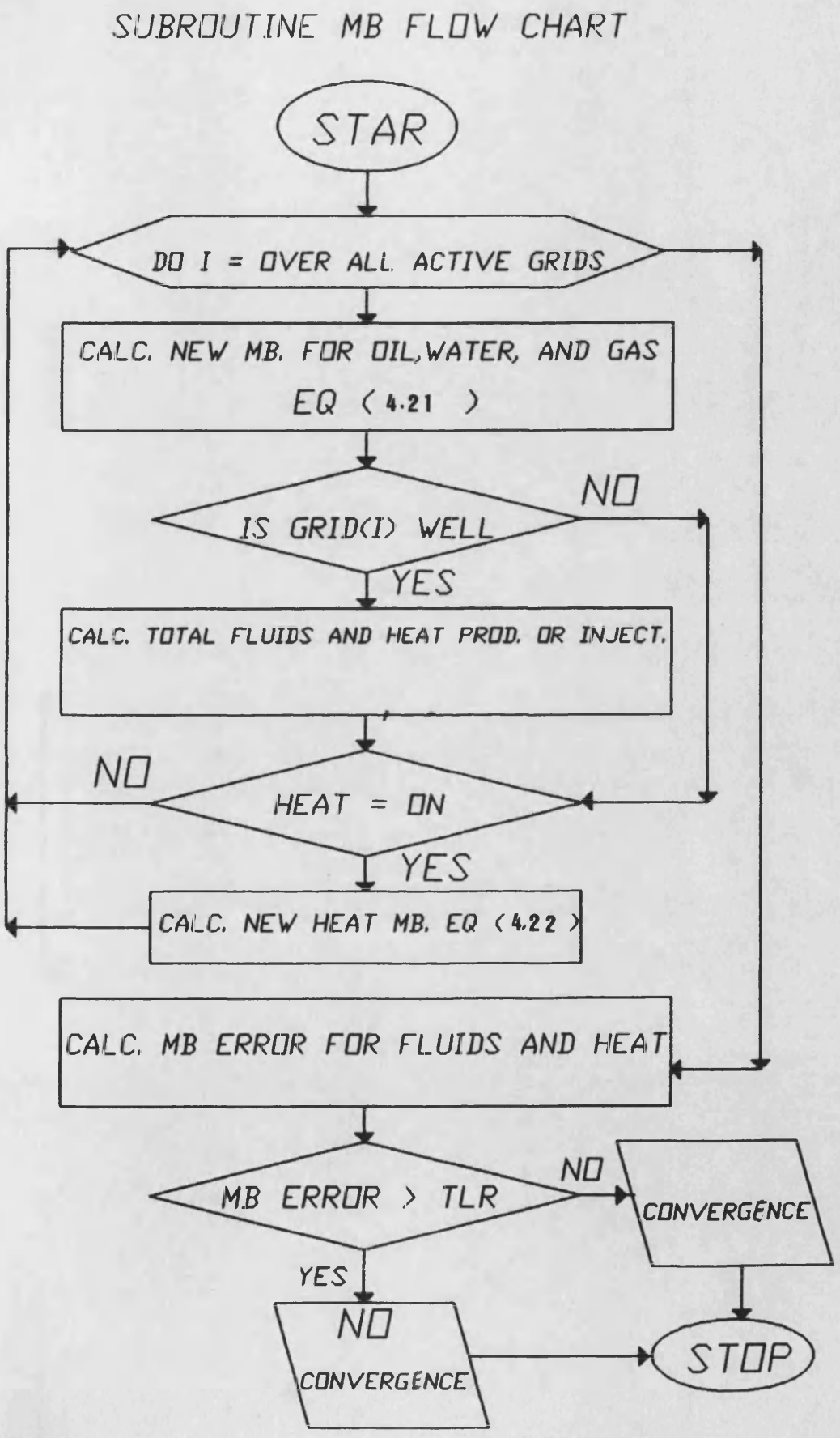


FIGURE 5.21 SUBROUTINE MB FLOW CHART

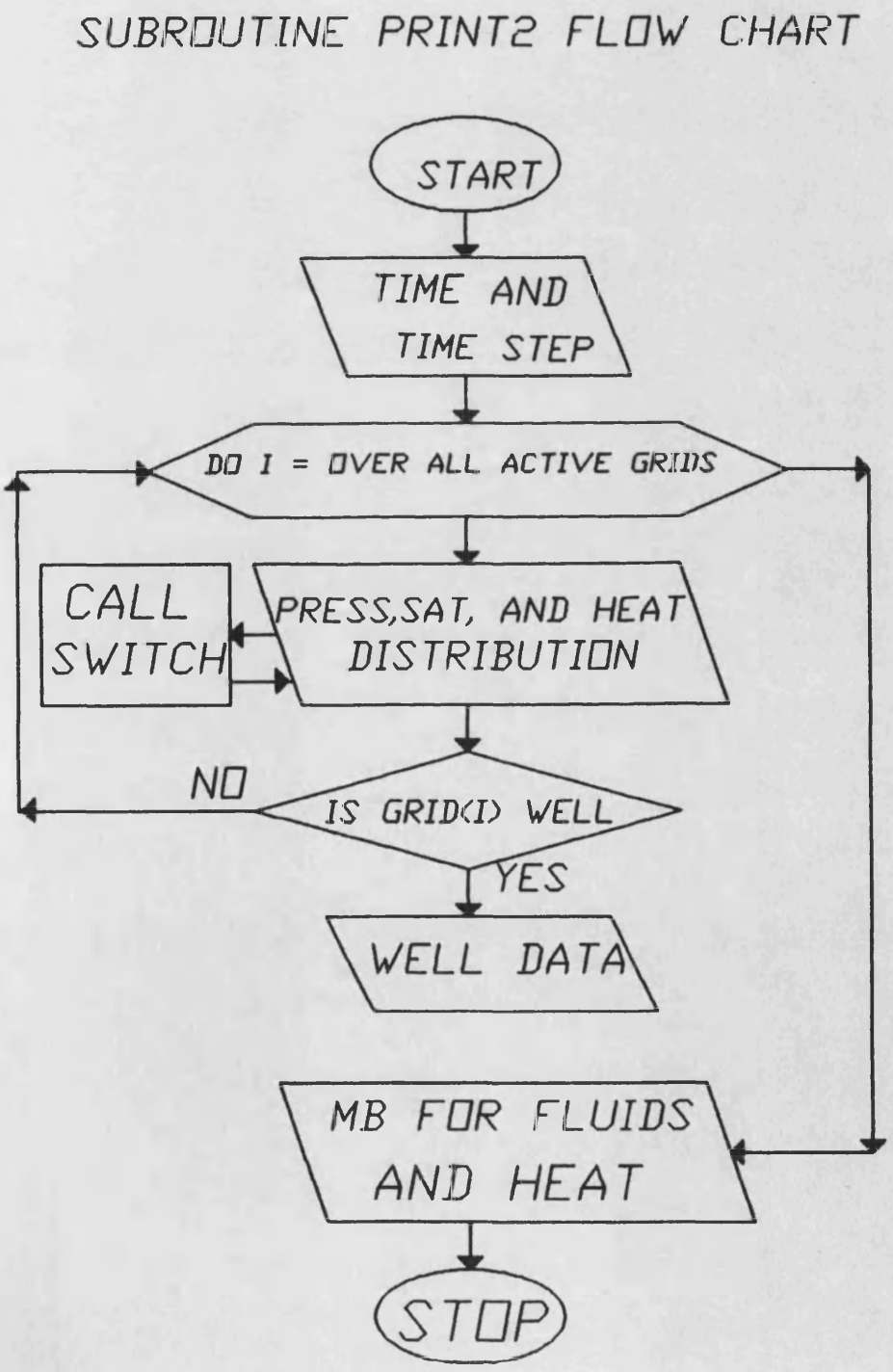


FIGURE 5.22 SUBROUTINE PRINT2 FLOW CHART

5.4 References

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6 - Knuth, D. E.: "The art of Computer programming." Addison-Wesley, (1973).

CHAPTER SIX

RESULTS AND CONCLUSIONS

CHAPTER SIX CONTENTS

FIGURES

TABLES

CHAPTER SIX
RESULTS AND CONCLUSIONS

6.1 Introduction

The model described in previous chapters has been translated into a computer programme written in Fortran 77 language. The programme was checked and run on a Honeywell 68 DPS computer.
With some simplification, however it is also possible to run the model on personal computer (PC) equipped with Fortran compiler and math co-processor.

Model checking was carried out at every step of the model development. In general, two types of checks were used: mathematical checks (for example, arithmetic, dimension and boundary conditions) and engineering-sense checks (for example, factors behave as expected). These checks were conducted to insure error free coding and loading ability.

6.2 Model validation

As has been explained in the earlier chapters, the energy equation utilizes the calculated pressure and

saturation distributions to solve for the temperature distribution. In order to ease the validation process the model was divided into two parts:

1. isothermal model.
2. thermal model.

6.2.1 Isothermal Model

To illustrate the model's ability to predict pressure and saturation distributions, the following hypothetical tests were conducted based on the data contained in appendix C. These data were chosen within an acceptable range of actual reservoirs conditions.

Case-1: zero source term

Although this test is very simple and the solution is easy to predict; nevertheless, the programme proceeded to calculate all the required matrices and converge to the solution. The data used in this test are listed in appendix C, Tables 6A-E. The initial conditions are given in Table 6.1a. Because the reservoir was not disturbed by production or injection, the model print out showed, as expected, that the initial pressure and saturation distribution were sustained during the simulator run.

TABLE 6.1A-CASE 1 INITIAL CONDITIONS				
PRESSURE	TEMPERATURE	SATURATION		
psi	*°F*	*OIL*	*WATER*	*GAS*
2000.0	100.0	0.60	0.30	0.10

Case- 2: Saturation and Delta Time Effects :

This run examines the model sensitivity to different delta times.

Selection of the optimum delta time is based upon the minimum error and variation between two time-steps or delta times. The time step error is defined as the error between two sequential time steps of the same delta time. This error is represented by two adjacent cells a cross the same delta time row in Tables 6.3A through 6.9A, and also in Figures 6.1 through 6.17. Where the delta time error is, the error between any two delta times for the same time step. This is described by any two cell in the same column in Tables 6.3A through 6.9A, and also in Tables 6.3B-C'.

Geometric, fluid and rock properties chosen for the sensitivity analysis are given in appendix C, Tables (6A-E). Where the initial condictions and well data are shown in the following Tables 6.2A and 6.2B.

TABLE 6.2A-CASE 2 WELL INPUT DATA							
WELL NO.	TIME (D)		LOCATION		RATE (BBL/D)		
	START	*END*	*I*	*J*	*OIL*	*WATER*	*GAS*
1	0.0	45.0	1	2	-50.0	0.0	0.0

TABLE 6.2B CASE 2 INITIAL CONDITIONS				
PRESSURE	TEMPERATURE	SATURATION		
		OIL	*WATER*	*GAS*
1900.0	100.0	0.55	0.40	0.05

Mass Balance errors:

Oil, water, and gas material balances are presented in Tables 6.3 and Figures 6.1 through 6.3. The material balance is an essential requisition for model convergence.

A. Oil Material Balance

From Table 6.3A, We notice that the smallest variation and magnification of oil material balance error is achieved with delta time (dt) 5.

TABLE 6.3A-OIL MATERIAL BALANCE						
t \ dt	5	10	15	20	25	30
5	1.0006	1.0006	1.0006	1.0006	1.0007	1.0007
10		1.0007		1.0007		1.0008
15			1.0011			1.0012
20				1.0014		
30						1.0025

Although the error do not differ significantly for different time increments, Figure 6.1 shows that a more stable profile exists for delta times 5 and 10. The largest Oil Material Balance Error (OMBE) occurs with delta time 30. But the OMBE (0.0025) of delta time 30 is still within the acceptable tolerance.

Further more, The difference between delta times MBE are demonstrated in Table 6.3B. Here, it can be seen that smallest error occurs between dt=5 and dt=10.

TABLE 6.3B–EFFECT OF DELTA TIME ON OIL MATERIAL BALANCE

dt & t / t	5	10	15	20	30
5	0.0	0.0001	0.0005	0.0008	0.0018
10		0.0		0.0007	0.0017
15			0.0		0.0013
20				0.0	
30					0.0

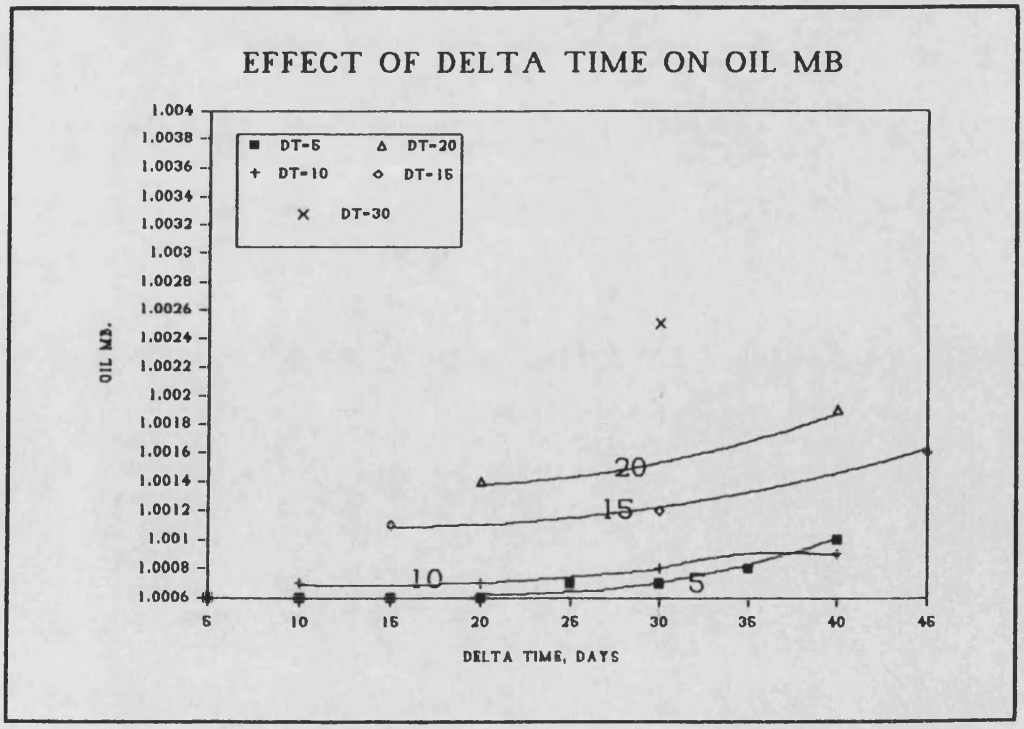

FIGURE 6.1 EFFECT OF DELTA TIME ON OIL MATERIAL BALANCE

B. Water Material Balance

TABLE 6.4A-WATER MATERIAL BALANCE						
dt / t	5	10	15	20	25	30
5	1.0002	1.0002	1.0002	1.0002	1.0002	1.0002
10		1.0002		1.0002		1.0002
15			1.0003			1.0002
20				1.0003		
30						1.0007

More consistent and lower error values are achieved for water material balance compared with the oil Material Balance as shown in Table 6.4A. IT should be mentioned that the production of water is only 1 bbl/d compared to oil production of 50 bbl/d.

Table 6.4B and figure 6.2 show the solution is more stable for dt= 5 and dt= 10.

TABLE 6.4B–EFFECT OF DELTA TIME ON WATER MATERIAL BALANCE

dt & t	5	10	15	20	30
5	0.0	0	0.0001	0.0001	0.0005
10	0	0.0		0.0001	0.0005
15			0.0		0.0004
20				0.0	
30					0.0

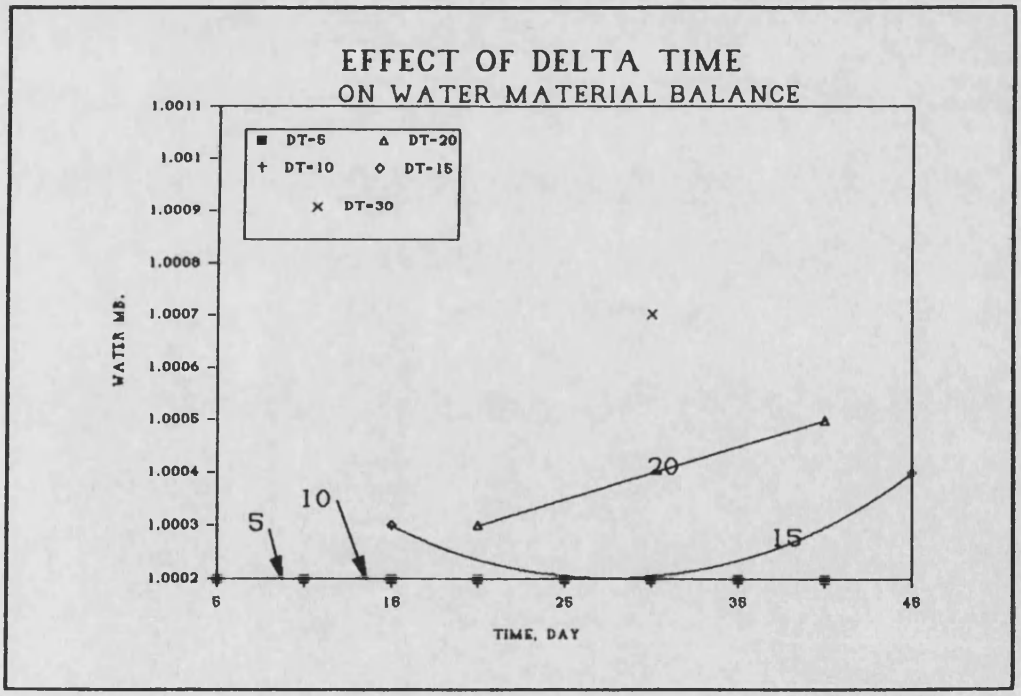

FIGURE 6.2 EFFECT OF DELTA TIME ON WATER MATERIAL BALANCE

C-Gas Material Balance

TABLE 6.5A-GAS MATERIAL BALANCE						
t \ dt	5	10	15	20	25	30
5	0.997	0.9966	0.9973	0.9975	0.9977	0.9979
10		1.0041		1.0048		1.0069
15			1.0054			1.0079
20				1.0073		
30						1.0074

Among the three phases, gas has the largest Material Balances Error, as shown in Table 6.5A and Figure 6.3. The smallest MB error is achieved for dt= 5, which is also the most stable. Figure 3 clearly shows that when dt> 5, there is letter to choose between these as regard solution accuracy.

TABLE 6.5B-EFFECT OF DELTA TIME ON GAS MATERIAL BALANCE					
dt & t / t	5	10	15	20	30
5	0	0.0075	0.0081	0.0098	0.0095
10		0		0.0025	0.0005
15			0		-0.0005
20				0	
30					0

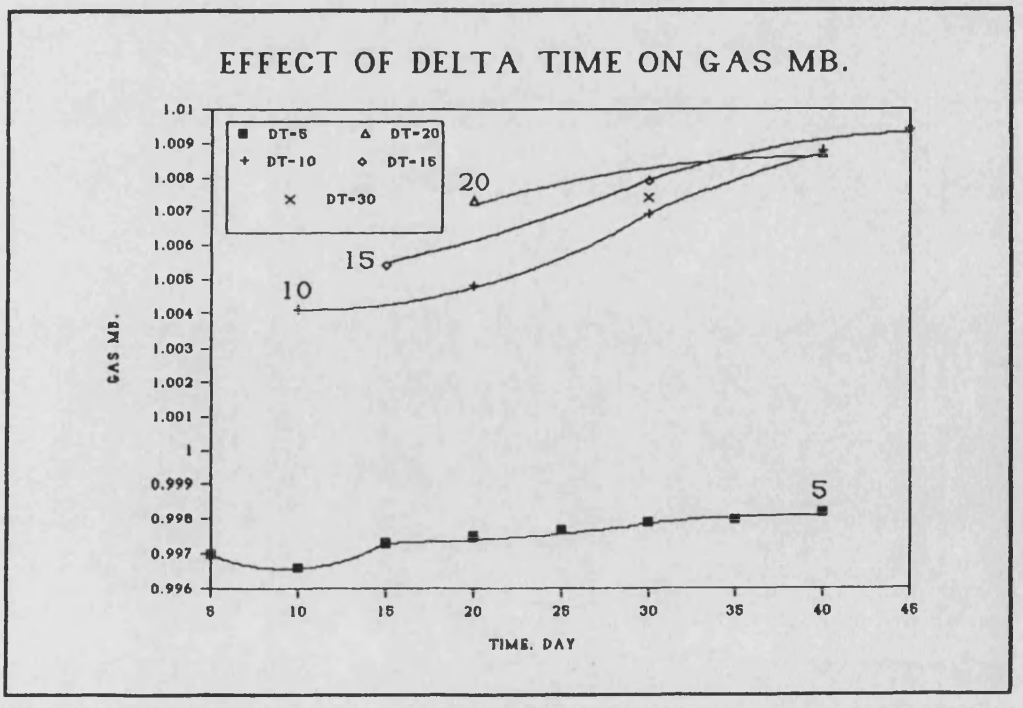

FIGURE 6.3 EFFECT OF DELTA TIME ON GAS MATERIAL BALANCE

Average pressure and saturations

The effect of time increment (dt) on the accuracy of average reservoir pressure, and oil, water and gas saturations is given in Tables 6.4 through 6.5c and Figures 6.4 through 6.7.

A-Pressure

TABLE 6.6A-PRESSURE						
dt / t	5	10	15	20	25	30
5	1872.9	1845.6	1817.9	1789.4	1759.6	1728.2
10		1858.1		1816.1		1770.4
15			1838.3			1771.9
20				1817.8		
30						1758.6

The decline in the pressure profile, shown in Table 6.6A and Figure 6.4, reflects the correct behavior of a reservoir under constant rate production conditions. For a delta time of 5, the decline is approximately equals 1.4% of the initial pressure per time step for an oil production of 250.0 BBl.

TABLE 6.6B-EFFECT OF DELTA TIME ON PRESSURE					
dt & t / t	5	10	15	20	30
5	0.0	12.5	20.4	28.4	30.4
10		0.0		1.7	-11.8
15			0.0		-13.3
20				0.0	
30					0.0

Table 6.6B shows noticeable difference between the calculated average reservoir pressure for different delta times. For example, there is 30.4 psi difference between dt= 5 and dt= 30 at time step 30. This difference is proportional to the difference between two delta times.

The most likely cause of average pressure difference is the approximation of the fluid properties gradients, for example formation volume factor gradient (dB/dP), which are reflected in the calculation of total compressiblity. This error is enlarge with increasing deltas time value.

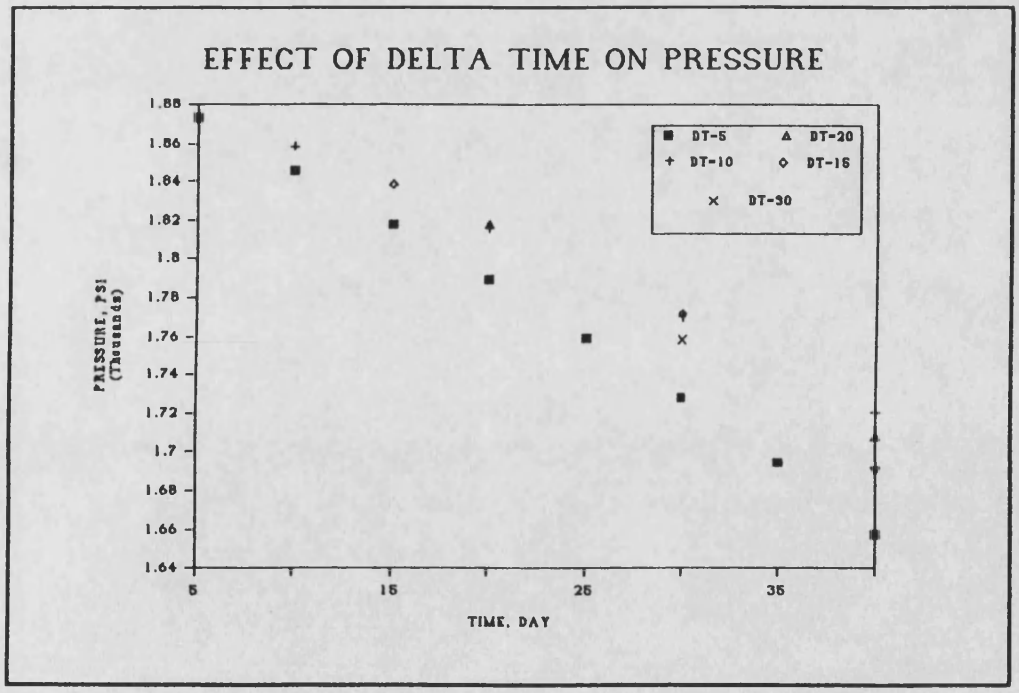

FIGURE 6.4 THE EFFECT OF DELTA TIME ON PRESSURE

B-Oil Saturation

TABLE 6.7A-OIL SATURATION						
t \ dt	5	10	15	20	25	30
5	0.5415	0.5331	0.5246	0.5162	0.5077	0.4992
10		0.5356		0.5211		0.5066
15			0.5283			0.5067
20				0.5211		
30						0.5067

With a production of 50 barrels per day, the decline in oil saturation, shown in Table 6.6A, is expected. The average oil saturation decline is approximately 1.55% of the initial oil saturation per time step as shown in Figure 6.5 and Table 6.7A.

Table 6.7B shows the the absolute values of oil saturation difference cause by the variation of delta time. These values range from 0.0 to 0.0075, which clearly demonstrate that the effect of large delta time on oil saturation is diminutive.

TABLE 6.7B-EFFECT OF DELTA TIME ON OIL SATURATION					
dt & t / t	5	10	15	20	30
5	0.0	0.0025	0.0037	0.0049	0.0075
10		0.0		0	0.0001
15			0.0		0
20				0.0	
30					0.0

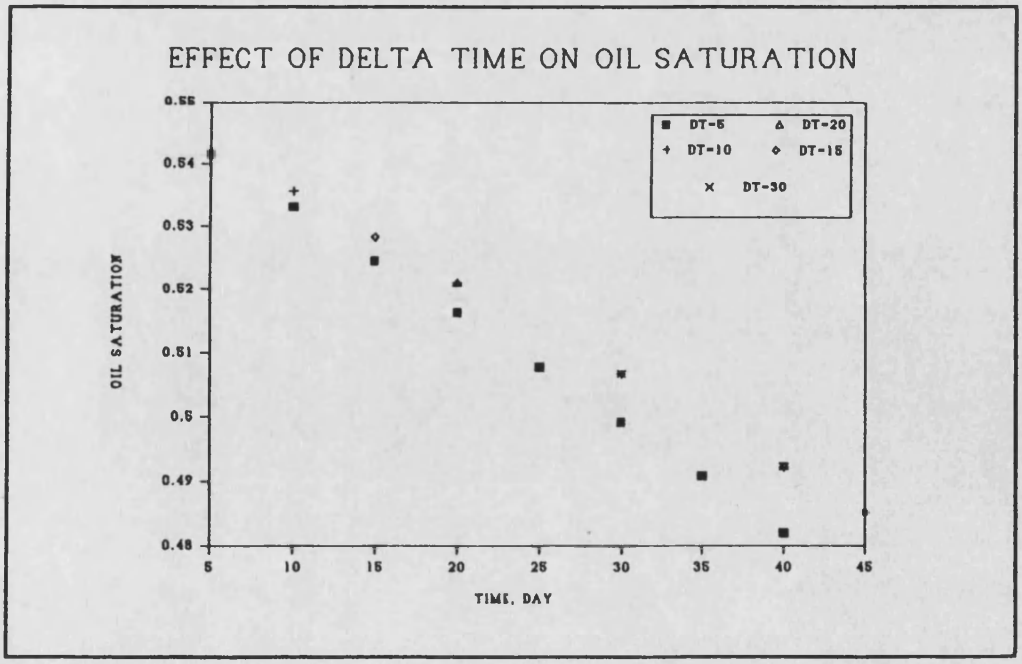

FIGURE 6.5 EFFECT OF DELTA TIME ON OIL SATURATION

C-Water Saturation

TABLE 6.8A-WATER SATURATION						
t \ dt	5	10	15	20	25	30
5	0.4	0.3999	0.3999	0.3999	0.3998	0.3998
10		0.4016		0.4032		0.4049
15			0.4024			0.405
20				0.4032		
30						0.4051

The well model output of the simulator, see Appendix D, shows that water production rate is only 1 barrel per day. Hence, the water saturation is not expected to show any significant decline.

At dt= 5 the water saturation exhibits a decline of only 0.025% relative to the initial water saturation as shown in Table 6.8A and Figure 6.6. Figure 6.6 shows higher delta times do not reflect the same trend of prediction.

TABLE 6.8B- EFFECT OF DELTA TIME ON WATER SATURATION					
dt & t / t	5	10	15	20	30
5	0.0	0.00161	0.00241	0.00321	0.0053
10		0.0		0	0.0002
15			0.0		0.0001
20				0.0	
30					0.0

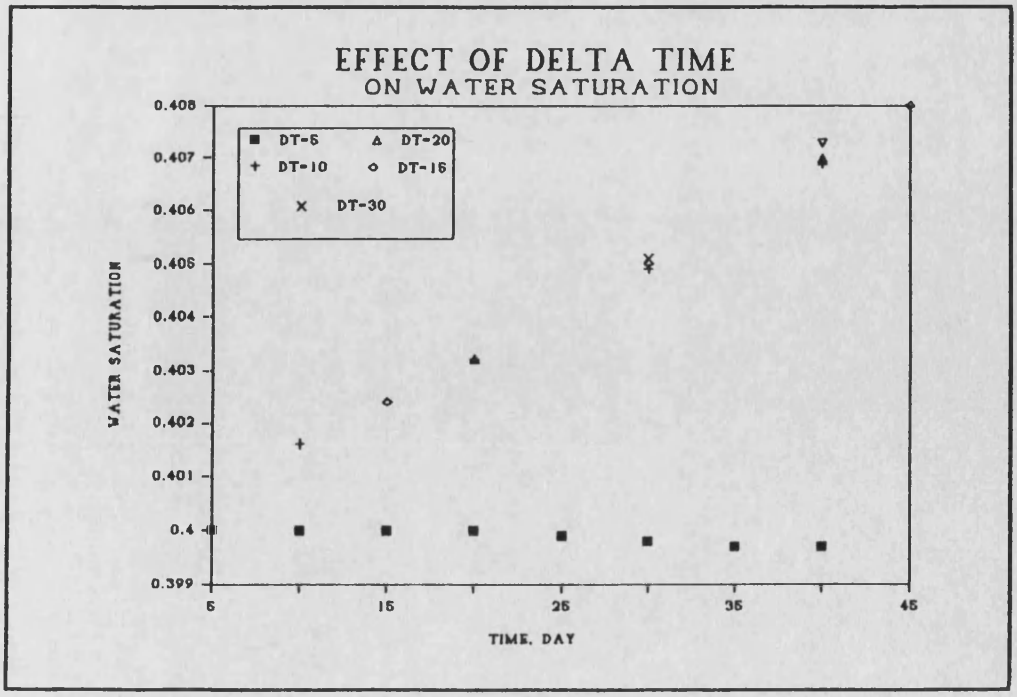

FIGURE 6.6 THE EFFECT OF DELTA TIME ON WATER SATURATION

D-Gas Saturation

TABLE 6.9A-GAS SATURATION						
dt / t	5	10	15	20	25	30
5	0.0585	0.0669	0.0754	0.0838	0.0924	0.101
10		0.0628		0.0757		0.0885
15			0.0693			0.0883
20				0.0757		
30						0.0882

The largest relative change in phase saturations occurs in the gas phase.

The increasing gas saturation trend, shown in Figure 6.7, reflect the evolved gas dissolution due to declining reservoir pressure. The average increment in the gas saturation increases by approximately 17% of the initial gas saturation per time step.

TABLE 6.9B-EFFECT OF DELTA TIME ON GAS SATURATIONE

dt & t	5	10	15	20	30
5	0.0	0.00411	0.07541	0.08381	0.101
10		0.0		0	0
15			0.0		0
20				0.0	
30					0.0

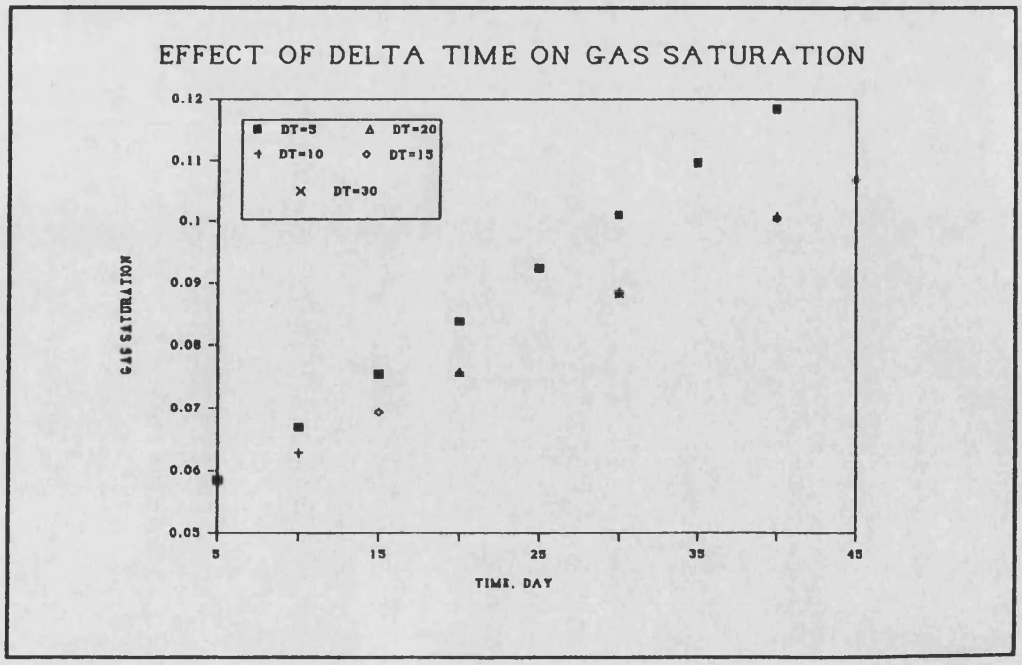

FIGURE 6.7 THE EFFECT OF DELTA TIME ON GAS SATURATION

If a comparison is made between the predicted error in the Material Balance and saturation, thus it is apparent that there is a positive correlation effect between the two quantities.

The variation of saturation is calculated as follows:

$$\frac{(S_{n+1} - S_n)}{S_n} \qquad (6.1)$$

Where 'i' is representing a time step number.

Figures 6.8 and 6.9 illustrate this relation, where the variation of saturation against time for dt= 5, is shown in Figure 6.8. The corresponding variation of material balance error versus time is shown in Figure 6.9. The most likely cause of this effect is the explicit treatment of saturation in the model.

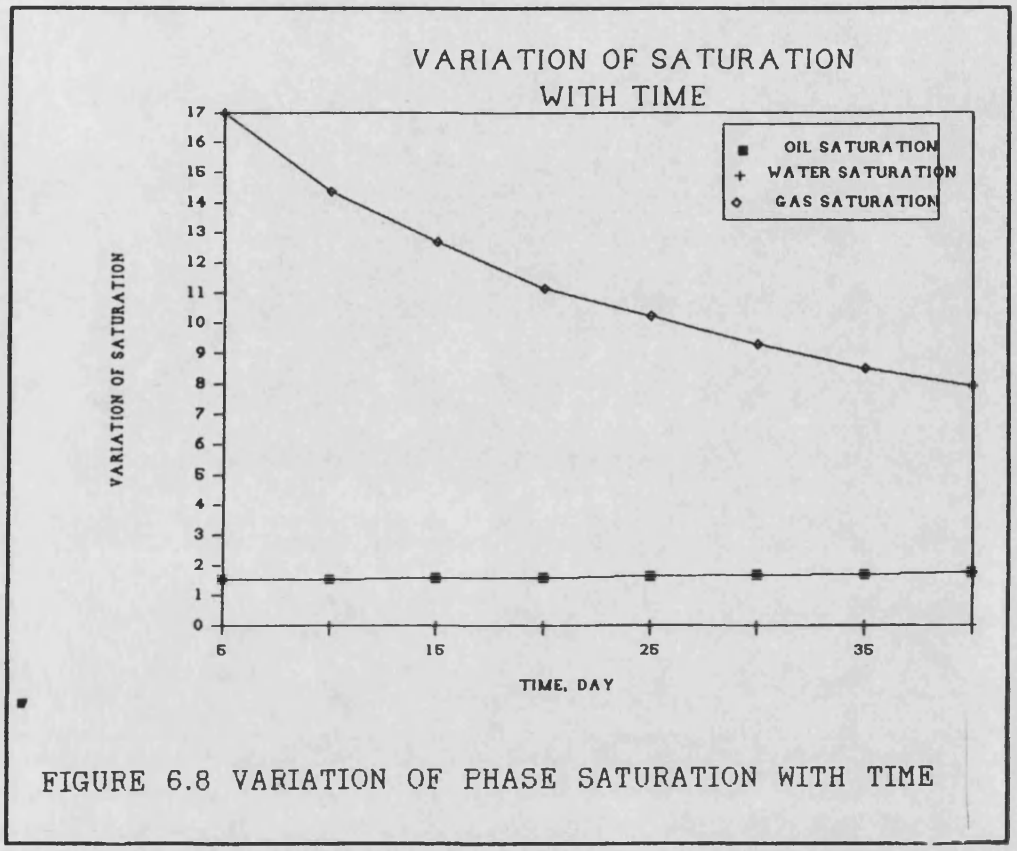

FIGURE 6.8 VARIATION OF PHASE SATURATION WITH TIME

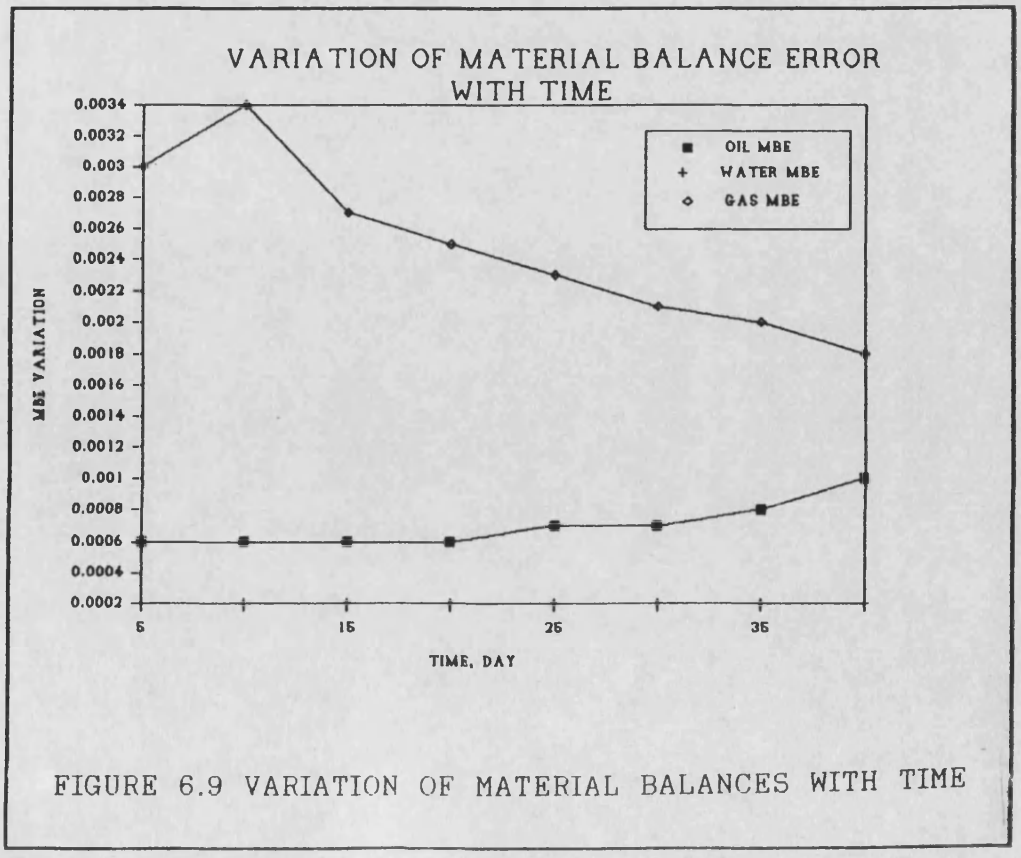

FIGURE 6.9 VARIATION OF MATERIAL BALANCES WITH TIME

Generally, the IMPES method of solution for an isothermal reservoir is satisfactory. However, large changes in saturation, such the 17% change per time step predicted previously for the gas saturation, could inevitably give arise to a very large error in the material balance, with consequent numerical instability problems.

As Tables 6.3B-9B previously, a delta time of 5 days yields the smallest material balance error and maximum stability. Accordingly, dt= 5 days is selected for the following simulator runs. It should be noticed , however, that larger delta times still produce results within an acceptable tolerance level.

Case- 3 Water Flood

This simulation run is designed to predict the behavior of a water flood for a rectangle-shaped reservoir, which has four corner producers and a centeral injector (Figuer 6.10). Well data for this case are shown in Table 6.10A, other data required are given in Appendix C.

TABLE 6.10A- CASE 3 WELL INPUT DATA							
WELL NO.	TIME (D)		LOCATION		RATE (BBL/D)		
	START	*END*	*I*	*J*	*OIL*	*WATER*	*GAS*
1	0.0	40.0	1	2	-50.0	0.0	0.0
2	0.0	40.0	1	5	-50.0	0.0	0.0
3	0.0	40.0	3	3	0.0	200.0	0.0
4	0.0	40.0	5	1	-50.0	0.0	0.0
5	0.0	40.0	5	5	-50.0	0.0	0.0

Part of the simulator output at time 10 and 20 days is showen below. The complete output is listed in Appendix D.

```
  PRODUCTION DATA AFTER  2  TIME STEPS
   TIME =    10.00    DAYS

       ***OIL PRESSURE DISTRIBUTION ****

   j=     1          2          3          4          5

I= 1   1894.44    1897.59    1898.77    1897.59   1894.44
       *******                                    ********
I= 2   1897.59    1899.25    1900.81    1899.25   1897.59

I= 3   1898.76    1900.81    1906.46    1900.81   1898.76
                              ^^^^^^^
I= 4   1897.59    1899.25    1900.81    1899.25   1897.59

I= 5   1894.44    1897.59    1898.77    1897.59   1894.44
       *******                                    *******
```

*** OIL SATURATION ARRAY ****

	j= 1	2	3	4	5
I= 1	0.5483	0.5493	0.5495	0.5493	0.5483
I= 2	0.5493	0.5498	0.5501	0.5498	0.5493
I= 3	0.5495	0.5501	0.5181	0.5501	0.5495
I= 4	0.5493	0.5498	0.5501	0.5498	0.5493
I= 5	0.5483	0.5493	0.5495	0.5493	0.5483

*** WATER SATURATION ARRAY ****

	j= 1	2	3	4	5
I= 1	0.4000	0.4000	0.4000	0.4000	0.4000
I= 2	0.4000	0.4000	0.4001	0.4000	0.4000
I= 3	0.4000	0.4001	0.4354	0.4001	0.4000
I= 4	0.4000	0.4000	0.4001	0.4000	0.4000
I= 5	0.4000	0.4000	0.4000	0.4000	0.4000

*** GAS SATURATION ARRAY ****

	j= 1	2	3	4	5
I= 1	0.0517	0.0507	0.0505	0.0507	0.0517
I= 2	0.0507	0.0502	0.0499	0.0502	0.0507
I= 3	0.0505	0.0499	0.0465	0.0499	0.0505
I= 4	0.0507	0.0502	0.0499	0.0502	0.0507
I= 5	0.0517	0.0507	0.0505	0.0507	0.0517

**** WELL REPORT ****

ROW I	COLUMN J	OIL RATE (STB/D)	WATER RATE (STB/D)	GAS RATE (SCF/D)	GOR (SCF/STB)
1	1	-50.0	-1.1	-766.1	15.3212
5	1	-50.0	-1.1	-766.1	15.3212
1	5	-50.0	-1.1	-766.1	15.3212
5	5	-50.0	-1.1	-766.1	15.3212
3	3	0.0	200.0	0.0	0.0000

OIL MATERIAL BALANCE = 1.0000
WATER MATERIAL BALANCE = 1.0000
GAS MATERIAL BALANCE = 0.9972

PRESSURE AVERAGE = 1898.4 PSI
OIL SATURATION AVERAGE = 0.5481
WATER SATURATION AVERAGE = 0.4014
GAS SATURATION AVERAGE = .0505
TEMPERATURE AVERAGE = 100.00 F
OIL RECOVER IN THIS TIME STEP = 0.1657

PRODUCTION DATA AFTER 4 TIME STEPS
TIME = 20.00 DAYS

OIL PRESSURE DISTR .BUTION *

	j= 1	2	3	4	5
I= 1	1892.66	1895.92	1897.15	1895.92	1892.66
	*******				*******
I= 2	1895.92	1897.68	1899.31	1897.68	1895.92
I= 3	1897.15	1899.30	1906.01	1899.30	1897.15
			^^^^^^^		
I= 4	1895.92	1897.68	1899.31	1897.68	1895.92
I= 5	1892.66	1895.92	1897.15	1895.92	1892.66
	*******				*******

*** OIL SATURATION ARRAY ****

	j= 1	2	3	4	5
I= 1	0.5477	0.5488	0.5490	0.5488	0.5477
	*******				*******
I= 2	0.5488	0.5493	0.5492	0.5493	0.5488
I= 3	0.5490	0.5493	0.4848	0.5493	0.5490
			^^^^^^^		
I= 4	0.5488	0.5493	0.5492	0.5493	0.5488
I= 5	0.5477	0.5488	0.5490	0.5488	0.5477
	*******				*******

*** WATER SATURATION ARRAY ****

	j= 1	2	3	4	5
I= 1	0.4000	0.4000	0.4000	0.4000	0.4000
	*******				*******
I= 2	0.4000	0.4000	0.4003	0.4000	0.4000
I= 3	0.4000	0.4003	0.4699	0.4003	0.4000
			^^^^^^		
I= 4	0.4000	0.4000	0.4003	0.4000	0.4000
I= 5	0.4000	0.4000	0.4000	0.4000	0.4000
	*******				*******

*** GAS SATURATION ARRAY ****

	j= 1	2	3	4	5
I= 1	0.0523	0.0512	0.0510	0.0512	0.0523
	*******				*******
I= 2	0.0512	0.0507	0.0505	0.0507	0.0512
I= 3	0.0510	0.0504	0.0452	0.0504	0.0510
			^^^^^^^		
I= 4	0.0512	0.0507	0.0505	0.0507	0.0512
I= 5	0.0523	0.0512	0.0510	0.0512	0.0523
	*******				*******

**** WELL REPORT ****

ROW I	COLUMN J	OIL RATE (STB/D)	WATER RATE (STB/D)	GAS RATE (SCF/D)	GOR (SCF/STB)
1	1	-50.0	-1.1	-761.2	15.2232
5	1	-50.0	-1.1	-761.2	15.2232
1	5	-50.0	-1.1	-761.2	15.2232
5	5	-50.0	-1.1	-761.2	15.2232
3	3	0.0	200.0	0.0	0.0000

OIL MATERIAL BALANCE =1.0000
WATER MATERIAL BALANCE =1.0000
GAS MATERIAL BALANCE =1.0000

PRESSURE AVERAGE =1896.8
OIL SATURATION AVERAGE =0.5463
WATER SATURATION AVERAGE =0.4028
GAS SATURATION AVERAGE =0.0509
TEMPERATURE AVERAGE =100.00
OIL RECOVERY IN THIS TIME STEP =0.1657

Because of the symmetrical nature of the reservoir geometry, a quadrant representation of the reservoir response is used to reflect the behavior of the reservoir. Referring to Figure 6.10, we draw lines (observation section) through selected key grids (A,B,C,D and E) in order to observe the changes of pressure and saturation in reservoir.

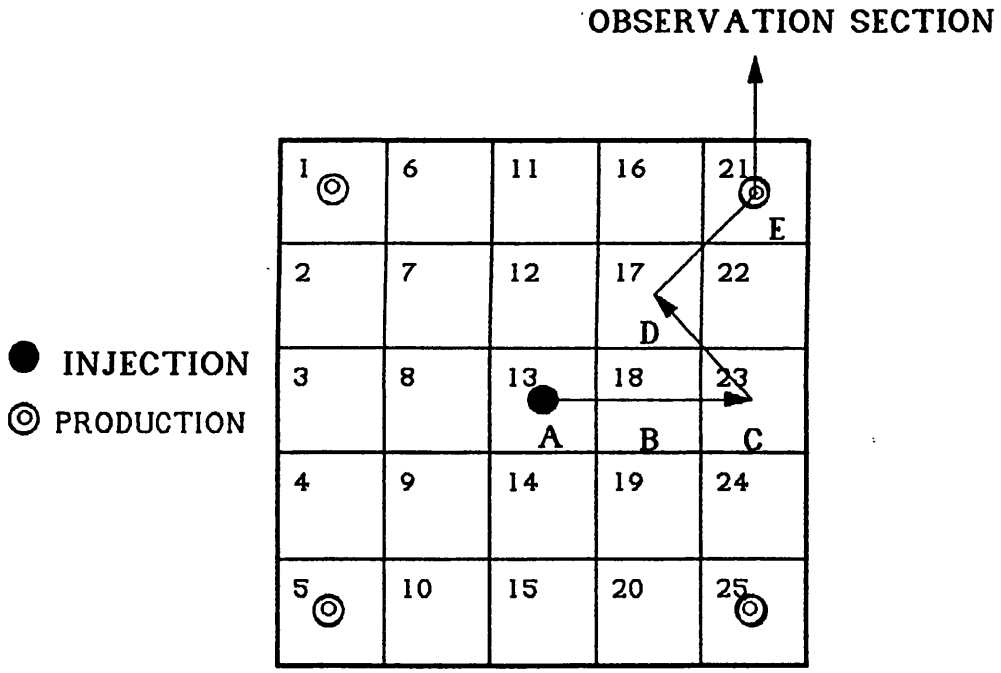

FIGURE 6.10- SECTION OF OBSERVATION THROUGH KEY GRIDS (A,B,C,D, AND E)

The model predictions of pressure and saturations at the key-grids are illustrated in Figures 6.11 through 6.14.

Pressure Distribution:

Initially, pressures at all the grids are the same and equal to 1900.0 psi. As water is injected into the reservoir the pressure begins to rise in the injector, and downstream the pressure declines in the direction of the producing well. At other grid location the pressure is proportional to the distance from the injector and the producer, as shown in Figure 6.11 and Table 6.11.

TABLE 6.11- PRESSURE DISTRIBUTION FOR KEY GRIDS						
	GRID KEY	A	B	C	D	E
TIME						
5		1905.7	1901.2	1899.5	1899.9	1895.9
10		1906.5	1900.8	1897.8	1899.3	1894.4
20		1906	1899.3	1897.2	1897.7	1892.7
30		1905.4	1897.7	1895.5	1896.1	1891
40		1904.9	1896.1	1893.9	1894.4	1889.4

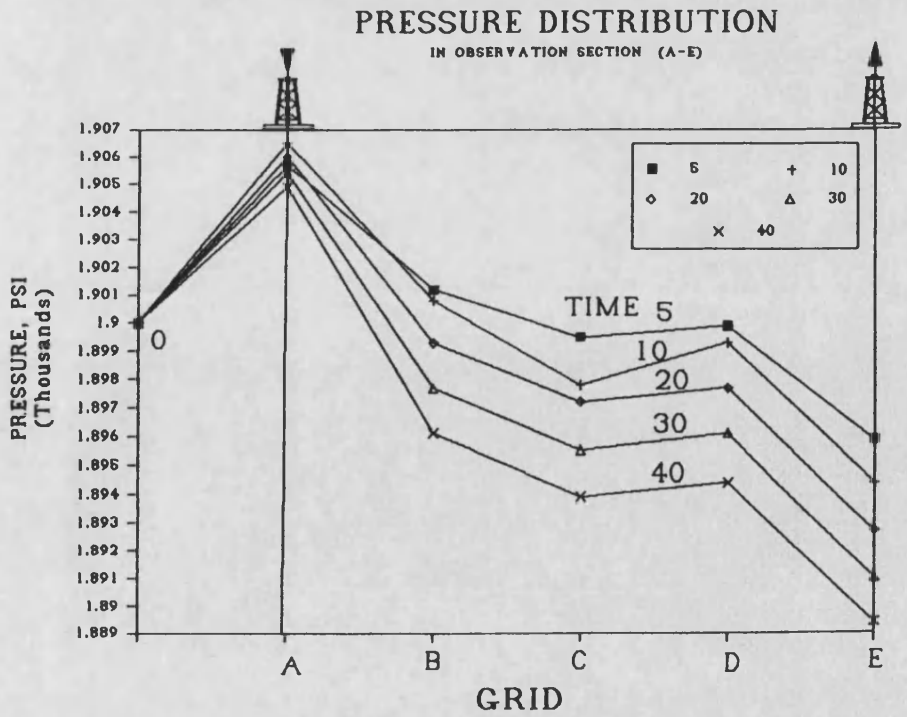

FIGURE 6.11 PRESSURE DISTRIBUTION

Saturations Distributions:

The initial oil, water and gas saturations are uniform through the reservoir at so= 0.55, sg= 0.40 and sw= 0.05 respectively (Table 6.2B).

As water injected into the reservoir, a water bank is created downstream in the reservoir leads to the development of an oil bank. The oil bank thus drives out the gas bank ahead of it[1]. Inspection of Table 6.13, and also the trends shown in Figure 6.13, shows a similar pattern of behavior predicted by the simulator. Thus, on commencement of water injection grid A exhibits an increase in water saturation (from 0.40 to 0.418), as shown in Figure 6.13 and Table 6.13. The increase in water saturation causes a sharp decrease in oil and gas saturations at grid A, which is shown in Figures 6.12-14 and Tables 6.12 and 6.14.

TABLE 6.12-OIL SATURATION FOR KEY GRIDS						
	KEY GRIDS	A	B	C	D	E
TIME						
5		0.5348	0.5503	0.5498	0.55	0.54
10		0.5181	0.5501	0.5495	0.5498	0.5483
20		0.4848	0.5493	0.549	0.5493	0.5477
30		0.4526	0.5481	0.5483	0.5488	0.5472
40		0.4219	0.5466	0.5478	0.5484	0.5467

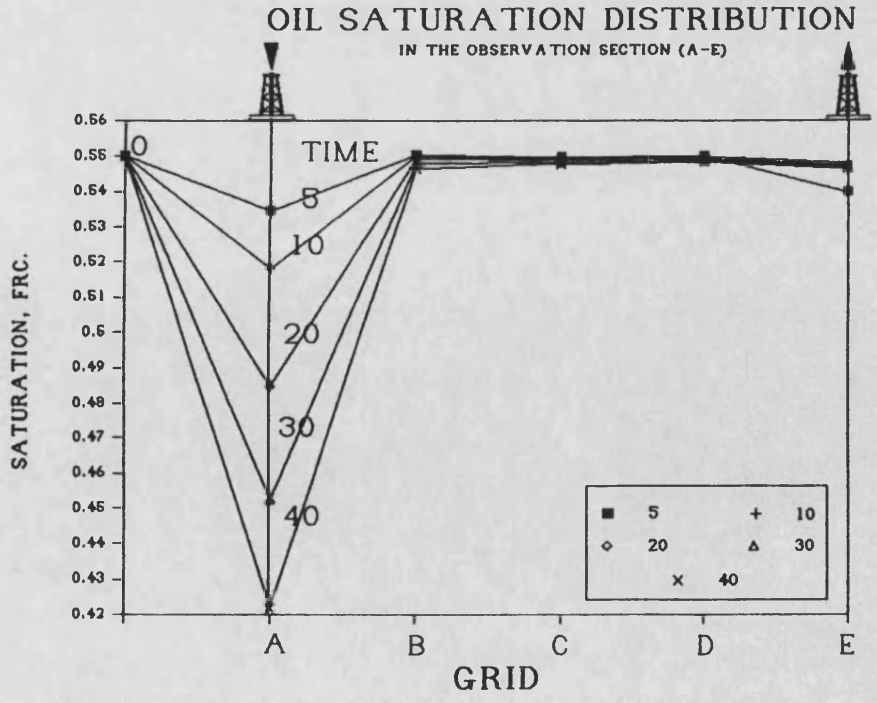

FIGURE 6.12 OIL SATURATION DISTRIBUTION

The high values of gas saturations at grids B, C, and D are caused by the gas bank advancing ahead of oil and water banks, due to the higher gas mobility ratio (gas-water), This is especially evident at times 5,10 and 20 days.

Dissolution of dissolved gas, as the pressure declines through the reservoir, causes an increase in the Gas-Oil ratio, giving rise to the increase gas saturation at 30 and 40 days.

In consequence, the advancing gas front restricts oil movement, as witnessed by the very small changes in oil saturation occurring at grids locations B,C, and D.

TABLE 6.13- WATER SATURTION FOR KEY GRIDS

	KEY GRIDS	A	B	C	D	E
TIME						
5		0.4178	0.4	0.4	0.4	0.4
10		0.4354	0.4001	0.4	0.4	0.4
20		0.4699	0.4003	0.4	0.4	0.4
30		0.5035	0.4008	0.4	0.4	0.4
40		0.5355	0.4017	0.4	0.4	0.4

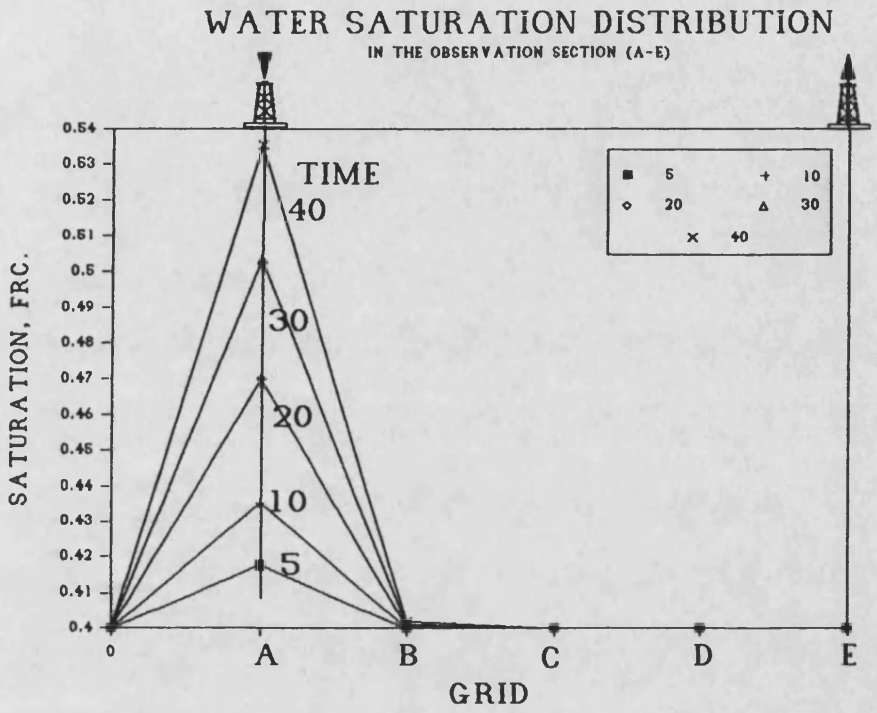

FIGURE 6.13 WATER SATURATION DISTRIBUTION

Grid E is a source grid, so the reduction in oil saturation is caused by oil production. Table 6.14 shows that grid E has the highest gas saturation. This high gas saturation is caused by reduction in in pressure below the grid initial pressure as shown in Figure 6.11, a higher gas-oil ratio is produce due to gas dissolution. This is the main reason for the increase in gas saturation at grid E.

TABLE 6.14- GAS SATURATION FOR KEY GRIDS						
	KEY GRIDS	A	B	C	D	E
TIME						
5		0.0474	0.0497	0.0502	0.05	0.0513
10		0.0465	0.0499	0.0505	0.0507	0.0517
20		0.0452	0.0504	0.051	0.0507	0.0523
30		0.0439	0.051	0.0516	0.0512	0.0528
40		0.0426	0.0517	0.0522	0.0516	0.0533

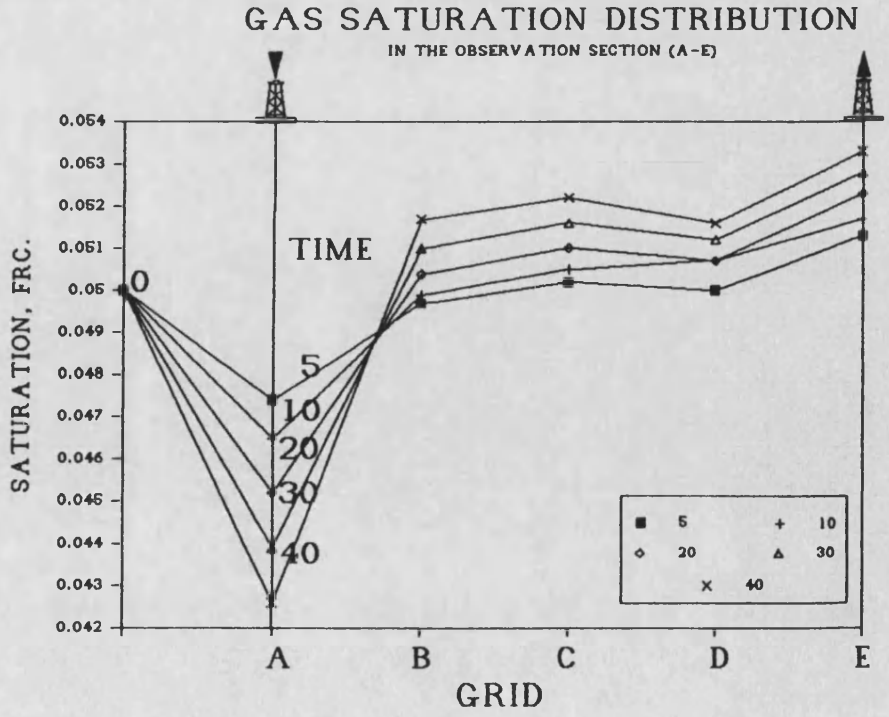

FIGURE 6.14 GAS SATURATION DISTRIBUTION

Excellent material balance results are obtained through the run. with zero oil material balance erro, and only 0.00003 error in the water material balance. The maximum material balance error occured for the gas phase but was still acceptable at 0.003 (Figure 6.15). In this case also, the error is affected by the variation in the phase saturation.

TABLE 6.15- MATERIAL BALANCE			
	OIL MB	WATER MB	GAS MB
TIME			
5	1	1.0001	1.0035
10	1	1	0.9972
20	1	1	1
30	1	1	1
40	1	1	1

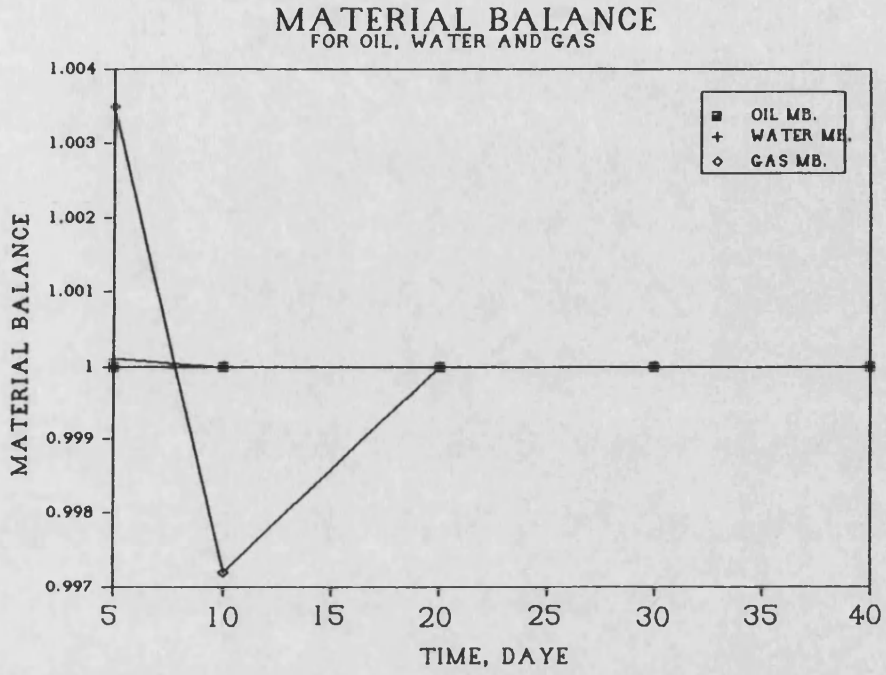

FIGURE 6.15 MATERIAL BALANCE

6.2.2 Thermal Model

Case- 4 Steam injection

The square reservoir configuration is used in this run. The input data are given in Appendix C.

In the initial work by Abdalla and coats[2] on a similar problem, there was good agreement between the experimental results and the simulator prediction. However, some doubt was expressed concerning the accuracy of the heat balance. It is important to

understand that the heat balance is not utilized as the means for checking the convergence of the solution.

In this work, the material balance and heat balance are used as the final checking condition for solution convergence, because correct pressure, saturation, and temperature distributions are necessary prerequisites to obtain accurate mass and heat balance values, as well as to maintain the model stability.

The resulting temperature distribution along the observation section (A-E) is represented graphically in Figure 6.16.

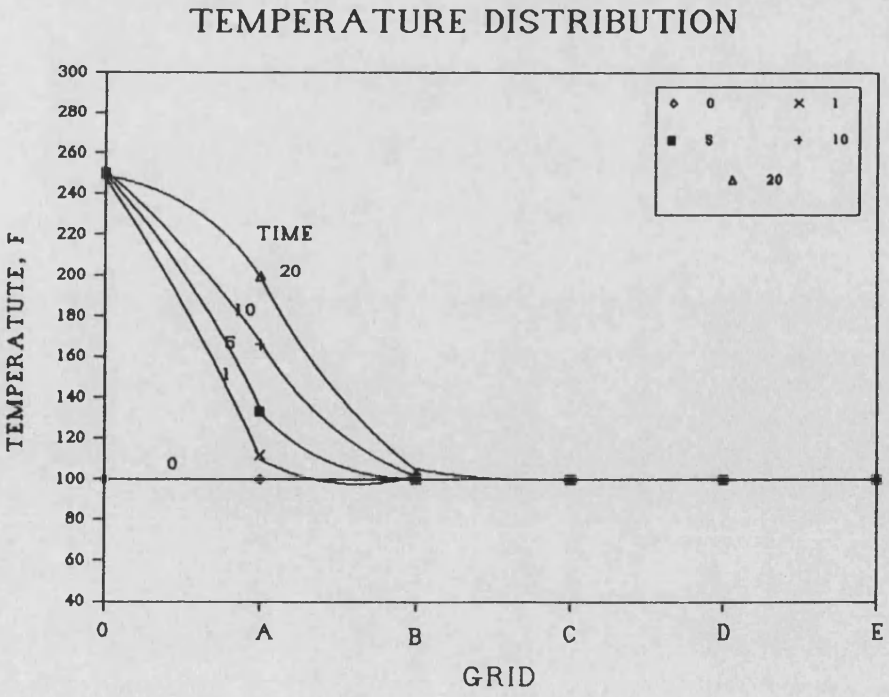

FIGURE 6.16 TEMPERATURE DISTRIBUTION

Although the temperature profile obtained was typical of the for steamed zone temperature, the solution method suffered from several difficulties. The method did not generally provide a satisfactory heat balance. In many instances the iteration method do not converge, even after a large number of iterations.

In spite of attempts of using various grid sizes and delta times to stabilize the model, the resultant heat balance errors were still unacceptably high. When using a small grid size the error incurred may impute a more rapid change in transmissibility between two adjacent grids. The IMPES method cannot handle this sufficiently well because of the explicit treatment of saturation.

In general the instability in the calculation method is attributed to the explicit treatment of saturation and condensation terms. In turn, explicit saturation contributes greatly to the error in the computation of transmissibility and source terms.

There are two possible causes of instability in the model due to the use of the IMPES method.

The first rises from the method of treatment of primary variables. The IMPES method treats the pressure implicitly, and it is therfor unconditionally stable. However, the explicit treatment of saturation introduces an instability conditions.
This is carried over to the energy equation (4.10a) by the velocities in the convection terms. The magnitude of the instability is further also increased by the saturation terms in the included on the right hand side of Equation (4.10a).

The second source of instabilit is introduce by the explicit treatment of the nonlinear terms, such as transmissibility, capillary pressure, relative permeability, and other reservoir properties. These variables are functions of pressure,temperature and saturation and other combination effect of these. Variables that depend on the saturation, for example, relative permeablity and capillary pressure, have a stronger nonlinearity effect than pressure or temperature dependent variables.

6.3 Conclusions

1. 2-D IMPES model which has been developed can efficiently simulate the flow of oil, water, and gas in an isothermal reservoir, under different production and injection conditions.

2. For the isothermal flow of fluid the an acceptable level of solution accuracy is achieved by the IMPES method when relatively large time increment are used. However, the use of smaller time increment can appreciably improved the solution accuracy.

3. The error in the material balance computation from the model shows a positive correlation effect with variation of saturation with time. If the rate of change of phase saturation becomes sufficiently high, due to high mobility of a phase or the use of small grid size, then the material balance error is greatly increased.

4. The IMPES method does not provide efficiently adequate method for solving the energy equation, and consequently a large error in the heat balance is generated. This is attributed mainly to the explicit treatment of phase

saturation, on which calculation of other transport effect depends. Stability further deteriorates when employing a non-uniform grid configuration.

6.4 REFERENCES

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CHAPTER SEVEN

SUGGESTIONS FOR FUTURE WORK

CHAPTER SEVEN CONTEN

CHAPTER SEVEN

SUGGESTIONS FOR FUTURE WORK

The quality of a simulation model depends on two areas, the application of transformation method, which transforms a differential equations into finite algebraic form, and the utilization of a digital computer (HardWare and SoftWare) to solve these algebraic equations. Accordingly the following suggestions and recommendations are based on the two areas:

. Transformation Method

. Computation

7.1 Transformation Method

The major sources of instability in the previous method is the explicit calculation of saturation term. To eliminate this cause the following formulation should be used:[1 2]

Oil phase

$$\Delta T_o\left(\Delta P_o - \gamma_o \Delta Z\right) + q_o = V_{sb}\frac{b}{\Delta t}\Delta_t\left(\frac{\phi S_o}{B_o}\right) \qquad (7.1a)$$

Water and Steam Phases:

$$\Delta T_w(\Delta P_w - \gamma_w \Delta Z) + \Delta T_s(\Delta P_s - \gamma_s \Delta Z) + q_w + q_s = \frac{V_b}{\Delta t}\Delta_t\left(\frac{\phi S_w}{B_w} + \frac{\phi S_s}{B_s}\right) \quad (4.10b)$$

Energy Equation

The partial differential equation for energy conservation is approximated in finite difference form as:

$$\Delta T_o H_o(\Delta P_o - \gamma_o \Delta Z) + \Delta T_w H_w(\Delta P_w - \gamma_w \Delta Z) + \Delta T_g H_g(\Delta P_g - \gamma_g \Delta Z) +$$
$$\Delta(K_h A \Delta T)_x + \Delta(K_h A \Delta T)_y + \Delta(K_h A \Delta T)_z + q_{ht} - q_{ht} =$$
$$\frac{V_b}{\Delta t}\Delta_t\{(\phi S_o \rho_o H_o + \phi S_w \rho_w H_w + \phi S_g \rho_g H_g) + (1-\phi)\rho_r C_r T\}$$

(4.10a)

Where equations 7.1 are subject to saturation, capillary pressures, and constant production constrains. The distributions of pressure, saturation and temperature are obtained by solving equations 7.1 simultaneously using direct or iterative methods.

Further the method requires $8IJ^2$ storage area (about 4 times more than IMPES method) as well as, $c(IJ^3 + 8IJ^3)$ more of computer work per step which is about 9 times more of IMPES method.

SEQ method requires equivalent specification of storage and work, but it suffers from limitation in the calculation of material balance error of the non-wetting phase. The error is proportional to the areal variation of Bw/Bn, where it can very large in case of gas-water system.

7.2 Computation:

Until now, almost all the large simulation studies were developed, tested and conducted exclusively on mainframe or minicomputers. because of speed and memory limitations of micro-computers. However, the new generation of micro-computer, based on 32-bit processor, includes numerous modifications and improvements over the earlier generation.

We suggest designing simulator workstation build around 32-microcomputers linked to mainframe or mini-computer. such parallel computer systems would provide a highly cost-effective and efficient alternative, as well as, an excellent environment for developing and testing simulator models.

Any proposed algorithms, or digital computer system design upon the treatment of partial differential equations, must be capable of performing the following functions [3]:

1. The acceptance of input data, include numerical information as well as coded instruction.

2. The storage of the input data, as well as, data generated in the course of the calculations.

3. The channeling and directing, within the computer, for data in such a manner that the proper calculations are preformed in sequence specified by the programme.

4. The performance of all required arithmetic and logical operations.

5. The readout of the solution of the problem in specified format.

The first modern electronic digital computer appeared in the late 1940s and early 1950s. The Van Newmenn architecture was quickly adopted as the result of the constrains imposed by the then-available electronics technology. It was not until the mid 1970s that there appeared classes of digital computers that differed from the Van Neumann architecture. Of particular importance in this connection was the introduction of the techniques of multiprocessing and the pipelining in digital computer system design.

The prospect of interconnect inexpensive microcomputer so as to achieve parallelism has been a tantalizing

anticipation for many years[4]. With the increasing availability of faster and larger memory 80386-based microcomputer, and the introduction of Transputers (from Thorn EMI, British), it becomes feasible to contemplate the instruction of multiprocessing computing system. The key is the configuration of the Transputer. Each transputer carries four serial links, which when plugged into other transputer, allows it to be assembled into its topology. This process achieves high computing speed and memory needed in the treatment of fluid and energy partial differential equation by finite difference method

7.3 REFERENCES

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APPENDIX A

IMPES METHOD

APPENDIX A
IMPES METHOD

IMPES Method

This method first proposed by Sheldon et al[1], and Stone and Gardner[2]. The IMPES method[3,4] utilise capillary pressure and saturation equations to obtain one multiphase single pressure equation from the following three single-phase equations.

oil phase :

$$\Delta T_o(\Delta P_o - \gamma_o \Delta Z) + q_o = \frac{V_b}{\Delta t}\Delta_t\left(\frac{\phi S_o}{B_o}\right) \qquad (A-1)$$

water phase :

$$\Delta T_w(\Delta P_w - \gamma_w \Delta Z) + q_w = \frac{V_b}{\Delta t}\Delta_t\left(\frac{\phi S_w}{B_w}\right) \qquad (A-2)$$

gas phase :

$$\Delta T_g(\Delta P_g - \gamma_g \Delta Z) + \Delta T_o R_{so}(\Delta P_o - \gamma_o \Delta Z) +$$
$$\Delta T_w R_{sw}(\Delta P_w - \gamma_w \Delta Z) + (q_g + g_o Rso + g_w Rsw) =$$
$$\frac{V_b}{\Delta t}\Delta_t\left(\frac{\phi S_g}{B_g} + \frac{\phi S_o R_{so}}{B_o} + \frac{\phi S_w R_{sw}}{B_w}\right) \qquad (A-3)$$

Capillary terms

$$P_{cw} = P_o - P_w \qquad (A-4)$$

$$P_{cg} = P_o + P_g \qquad (A-5)$$

where P c in equations (A-4) and (A-5) represents capillary pressure of water-oil and gas-oil system respectively.

The R.H.S of equations (A-1) and (A-2) can be expanded as:

$$\frac{V_b}{\Delta t}\Delta_t\left(\frac{\phi S_f}{B_f}\right) = \frac{V_b}{\Delta t}\left\{\left(\frac{\phi}{B_f}\right)\Delta_t(S_f) + \left[\phi^{n+1} S_f^n\left(\frac{1}{B_f}\right)^{\cdot} + \left(\frac{S_f}{B_f}\right)^n \phi^{\cdot}\right]\Delta_t(P_f)\right\} \qquad (A-6)$$

Where

$$\left(\frac{1}{B_f}\right)^{\cdot} = \frac{d\left(\frac{1}{B_f}\right)}{dP_f}$$

$$(\phi)^{\cdot} = \frac{d\phi}{dP_o}$$

The R.H.S of equation (4.1c) can be divided into three parts . these parts are :

$$\frac{V_b}{\Delta t}\Delta_t\left(\frac{\phi S_g}{B_g}\right) \qquad \text{part 1}$$

$$\frac{V_b}{\Delta t}\Delta_t\left(\frac{\phi S_o R_{so}}{B_o}\right) \qquad \text{part 2}$$

$$\frac{V_b}{\Delta t}\Delta_t\left(\frac{\phi S_w R_{sw}}{B_w}\right) \qquad \text{part 3}$$

Let us look at each part individually :

Part 1 :

$$\frac{V_b}{\Delta t}\Delta_t\left(\frac{\phi S_g}{B_g}\right) = \frac{V_b}{\Delta t}\left\{\left(\frac{\phi}{B_g}\right)\Delta_t(S_g) + \left[\phi^{n+1} S_g^n\left(\frac{1}{B_g}\right)' + \left(\frac{S_g}{B_g}\right)^n \phi'\right]\Delta_t(P_g)\right\} \qquad (A-7)$$

Part 2 :

$$\frac{V_b}{\Delta t}\Delta_t\left(\frac{\phi S_o R_{so}}{B_o}\right) = \frac{V_b}{\Delta t}\left\{\left(\frac{\phi R_{so}}{B_o}\right)\Delta_t(S_o) + \left[(\phi R_{so})^{n+1} S_o^n\left(\frac{1}{B_o}\right)' + \left(\frac{S_o R_{so}}{B_f}\right)^n \phi' + \phi^{n+1}\left(\frac{S_o}{B_o}\right)^n (R_{so})'\right]\Delta_t(P_o)\right\} \qquad (A-8)$$

Part 3 :

$$\frac{V_b}{\Delta t}\Delta_t\left(\frac{\phi S_w R_{sw}}{B_w}\right) = \frac{V_b}{\Delta t}\left\{\left(\frac{\phi R_{sw}}{B_w}\right)\Delta_t(S_w) + \left[(\phi R_{sw})^{n+1} S_w^n\left(\frac{1}{B_w}\right)' + \left(\frac{S_w R_{sw}}{B_f}\right)^n \phi' + \phi^{n+1}\left(\frac{S_w}{B_w}\right)^n (R_{sw})'\right]\Delta_t(P_w)\right\} \qquad (A-9)$$

Multiplying the oil equation by $(B_o+R_{sw}B_g)^{n+1}$, the water equation by $(B_g+R_{ws}B_g)^{n+1}$, the gas equation by B_g^{n+1} , substitute pw and pg by Equations (A-4) and (A-5). Adding these together we obtain the following IMPES equation :

$$(B_o-B_gR_{so})^{n+1}[\Delta T_o(\Delta P_o-\gamma_o\Delta Z)+q_o]+(B_w-B_gR_{ws})^{n+1}[\Delta T_w(\Delta P_o-\Delta P_{cw}-\gamma_o\Delta Z)+q_w]+$$
$$(B_g)^{n+1}[\Delta T_g(\Delta P_o-\Delta P_{cg}-\gamma_g\Delta Z)+\Delta[T_oR_{so}(\Delta P_o-\gamma_o\Delta Z)]+$$
$$\Delta[T_wR_{sw}(\Delta P_w-\Delta P_{cw}-\gamma_w\Delta Z)]+(q_g+g_oR_{so}+q_wR_{sw})=$$
$$\frac{V_b}{\Delta t}[(C_1o+C_2o+C_1w+C_2w+C_1g+\phi')\Delta_tP_o+(C_1w+C_2w+C_1g+(S_w)\phi')\Delta_tP_{cw}+$$
$$(C_1g+S_g\phi')\Delta P_{cg}] \qquad (A-10)$$

Where the coefficients C are:

$$C_1=(S_f)^n(\phi B_f)^{n+1}\left(\frac{1}{B_f}\right)' \qquad f = o,w,g$$

$$C_2=(S_f)^n\left(\frac{\phi B_g}{B_f}\right)^{n+1}(R_f)' \qquad f= o,w$$

The resulting Equation (A-10) contains only oil pressure, capillary pressure, and hydrostatic head terms. The equation can be written as

$$TP^{n+1}=D\left(P^{n+1}-P^n\right)+G+Q$$

Where T is the transmissibility matrix, D is the accumulation matrix, G is the gravity and capillary vector, and Q is the source vector.

If the IMPES equation is written for all grids in the x-y directions , where the standard ordering system is used , the following pentadiagonal matrix form is obtained :

$$\overline{A} = \begin{pmatrix} b_1 & c_1 & & f_1 & & & & & & \\ a_2 & b_2 & c_2 & & f_2 & & & & & \\ & \cdot & \cdot & \cdot & & \cdot & & & & \\ e_4 & & a_4 & b_4 & c_4 & & f_4 & & & \\ & \cdot & & \cdot & \cdot & \cdot & & \cdot & & \\ & & \cdot & & \cdot & \cdot & \cdot & & \cdot & \\ & & & \cdot & & \cdot & \cdot & \cdot & & \cdot \\ & & & & e_{M-1} & & a_{M-1} & b_{M-1} & c_{M-1} & \\ & & & & & e_M & & a_M & b_M & \end{pmatrix}$$

After solving the matrix by the use of direct or iterative algorithms, e.g., Gaussian elimination, ADIP, the new pressures distribution is obtained, the oil and water saturations are computed explicitly by substituting the calculated pressure in Equationas (A-1) and (A-2). Where the gas saturation is calculated by the following equation

$$S_g^{n+1} = 1 - S_o^{n+1} - S_w^{n+1}$$

Figure A-1 shows the a summary of IMPES method.

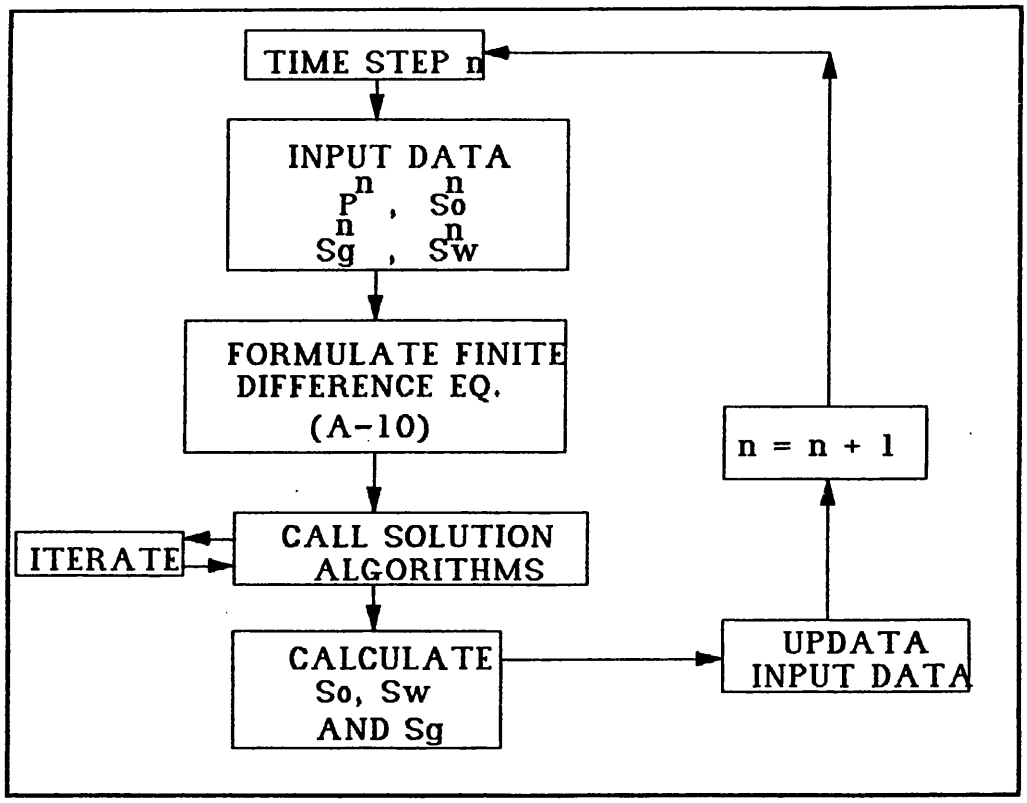

FIGURE A-1 IMPES METHOD SUMMARY

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APPENDIX B
COMPUTATIONAL LINEAR ALGEBRA

APPENDIX B
COMPUTATIONAL LINEAR ALGEBRA

The transformation of fluid and energy partial differential equations into finite difference form leads to large systems of simultaneous equations. These equations may be written as:

$$\overline{A}u = d \qquad (B-1)$$

Where:

u = represents the unknown vector (for example pressure, saturation, or temperature

$\overline{A}$ = coefficient matrix

d = right hand-side vector

The properties of coefficient matrix A depend not only on the nature of the problem involves or on the partial differential equations and assumptions used to represent the problem mathematically but also by the discretization procedure used to represent the partial differential equations.

In our solution A takes a sparse band matrix form, where its elements are real numbers. More specific, matrix A has tridiagonal form for one-dimensional grid as shown in Figure B-1.

$$A = \begin{pmatrix} b_1 & c_1 & & & \\ a_2 & b_2 & c_2 & & \\ & \cdot & \cdot & \cdot & \\ & & \cdot & \cdot & \cdot \\ & & & a_M & b_M \end{pmatrix}$$

FIGURE B-1 TRIDIAGONAL MATRIX

, or pentadiagonal for two-dimensional system as shown in Figure B-2.

$$A = \begin{pmatrix} b_1 & c_1 & & f_1 & & & & & \\ a_2 & b_2 & c_2 & & f_2 & & & & \\ & \cdot & \cdot & \cdot & & \cdot & & & \\ e_4 & & a_4 & b_4 & c_4 & & f_4 & & \\ & \cdot & & \cdot & \cdot & \cdot & & \cdot & \\ & & \cdot & & \cdot & \cdot & \cdot & & \cdot \\ & & & \cdot & & \cdot & \cdot & \cdot & \\ & & & & e_{M-1} & & a_{M-1} & b_{M-1} & c_{M-1} \\ & & & & & e_M & & a_M & b_M \end{pmatrix}$$

FIGURE B-2 PENTADIAGONAL MATRIX

It is also known that for this type of solution matrix A is characterised as diagonal dominate matrix.

The efficiency of a simulator depends on the its ability to solve the generated matrices with minimum computer time and storage requirements. There are vast a mount of publication in Engineering, mathematical, and computer science for solving systems of simultaneous equations. Essentially all these solutions are divided into two different classes of methods:

I. Direct Method

II. Iterative method

I-Direct Method

In a direct methods, the solution of a system of equations is obtained upon the completion of fixed number of steps and operations that predetermined in an exact manner. Some example of direct method are:

. Cramer's rule

. Gaussian elimination

. Gauss-Jordan method

. Matrix Decomposition (or LU Factorization)

1- LU Factorization

Among the above methods, LU Factorization is the most widely used method in reservoir simulation. The method also named "Crout reduction" or ,after anther discoverer, "Cholesky". The idea of this method is to transform or decomposed the coefficient matrix A into the product of lower triangular (L) and an upper triangular matrix (U) with 1's on its diagonal as shown in the figure B-3.

$$\overline{A} = LU \qquad (B-2.1)$$

$$L = \begin{pmatrix} l_{11} & 0 & & & & \\ l_{21} & l_{22} & 0 & & & \\ & \cdot & \cdot & \cdot & & \\ & & \cdot & \cdot & \cdot & \\ & & & \cdot & \cdot & \cdot \\ & & & & l_{nn-1} & l_{nn} \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & u_{12} & u_{22} & \cdot & \cdot & u_{1n} \\ 0 & 1 & u_{23} & \cdot & \cdot & u_{2n} \\ & 0 & \cdot & \cdot & & \\ & & \cdot & \cdot & \cdot & \\ & & & \cdot & \cdot & \cdot \\ & & & \cdot & \cdot & 1 \end{pmatrix}$$

FIGURE B-3 L AND U MATRICES

Where l and u elements are calculated by the following equations:

$$l_{ij} = 0 \qquad \text{if } i < j$$

$$u_{ij} = 0 \qquad \text{if } i > j$$

$$u_{ii} = 1 \qquad \text{if } i = j$$

Othere elements are then

$$l_{im} = a_{im} - \sum_{k=1}^{m-1} l_{ik} u_{km} \qquad \text{for } i = m, m+1, \ldots, n$$

$$m = 1, 2, \ldots, n$$

$$(B-3.1)$$

$$u_{mj} = \frac{1}{l_{mm}} \left(a_{mj} - \sum_{k=1}^{m-1} l_{mk} u_{kj} \right) \qquad \text{for } j = m+1, m+2, \ldots, n$$

$$m = 1, 2, 3, \ldots, n$$

$$(B-3.2)$$

There for equation (B-1) can be written as

$$LUu = d \qquad (B-3.3)$$

if we let

$$Y = L^{-1} d \qquad (B-3.4)$$

Then the solution is

$$u = YU^{-1} \qquad (B-3.5)$$

2- Thomas algorithm

The special form of matrix arising from the application of finite difference equation to one-dimensional problem, call tridiagonal matrix. Tridiagonal matrix has nonzero elements only on the main diagonal and the two adjacent to the main

diagonal. The form of this matrix is shown above in figure ( ).

The solution of Thomas algorithm to such matrix involves the following steps:

$$W_i = b_i - \frac{a_i c_{i-1}}{W_{i-1}} \qquad \text{with} \quad W_1 = b_1 \tag{B-4.1}$$

$$Q_i = d_i - \frac{a_i Q_{i-1}}{W_i} \qquad \text{with} \quad Q_1 = \frac{d_1}{b_1} \tag{B-4.2}$$

Back substitution ...

$$u_M = Q_M \tag{B-4.3}$$

$$u_i = Q_i - \frac{c_i u_{i+1}}{W_i} \tag{B-4.3}$$

II- Iterative Method

The iterative methods are an approximation numerical methods for solving system of equations have the form of equation (B-1). The idea upon which iterative methods are based is making an initial guess v as to the solutions U. These guess are then substituted in equation (B-1), resulted in new values of U. By repeating this procedure for a sufficiently large number of time , it converges to a predefined tolerance and resulted values are accepted as the solution to the finite difference equations. Iterative methods are most popular method used in reservoir simulation, primarily because they required less storage than direct method.

The simulator uses the following iterative methods:

1. Point Successive Overrelaxation (PSOR)
2. Line Successive Overrelaxation (LSOR)
3. Alternative Direction Implicit (ADI)

1- Point Successive Overrelaxation method (PSOR)

The PSOR method begin by rearrangement of a finite difference equations of each grid as follows:

$$O_M = \frac{1}{b_M}\left(d_M - e_M U_N^{K+1} - a_M U_W^{K+1} - c_M U_E^K - f_M U_S^K\right)$$

(B-5.1)

Then O relaxed by the following equation:

$$U_M^{K+1} = U_M^k - \omega\left(O_M - U_M^K\right)$$

(B-5.2)

Where:

ω = relaxation parameter

k = iteration level

This iterative process is continue until the following convergence condition is satisfied :

$$U^{k+1} - U^k = r \qquad (B-5.3)$$

2- Line Successive Overrelaxation method (LSOR)

This method is used to relax one line at a time, a column or row. The method is illustrated by the following equations:

$$a_M O_W + b_M O_M + c_M O_E + = \ddot{d}_M$$

(B-6.1)

Where:

$$\ddot{d}_M = d_M - e_M U_M^{k+1} - f_M U_M U_E^k \qquad (B-6.2)$$

Equation (B-6.1) generats a tridiagonal matrix which can be solved by Thomas algorithm. The value of θ can be relaxed by the following equation:

$$U_M^{k+1} = U^k - \omega(\theta_M - U_M^K) \qquad (B-6.3)$$

Relaxation Parameter

The successive overrelaxation (SOR) methods efficiency is strongly dependent upon the selection of the relaxation parameter ω. Therefor great care should be placed in the choice of this parameter. Unfortunately there is no certain way to obtain the optimum value of relaxation parameter for the real world problems (Varga shows method for calculating $\omega_{optimum}$ analytically for some regular shapes boundary). Trial-and-error procedure is used, which involve the variation of ω against the number of iterations required for convergence. The plot of these data generates typical shape as shown in figure (B-1). The optimum value of ω is located at the lowest point of the curve.

PSOR AND LSOR RELAXATION PARAMETER
BASED ON ITERATIONS

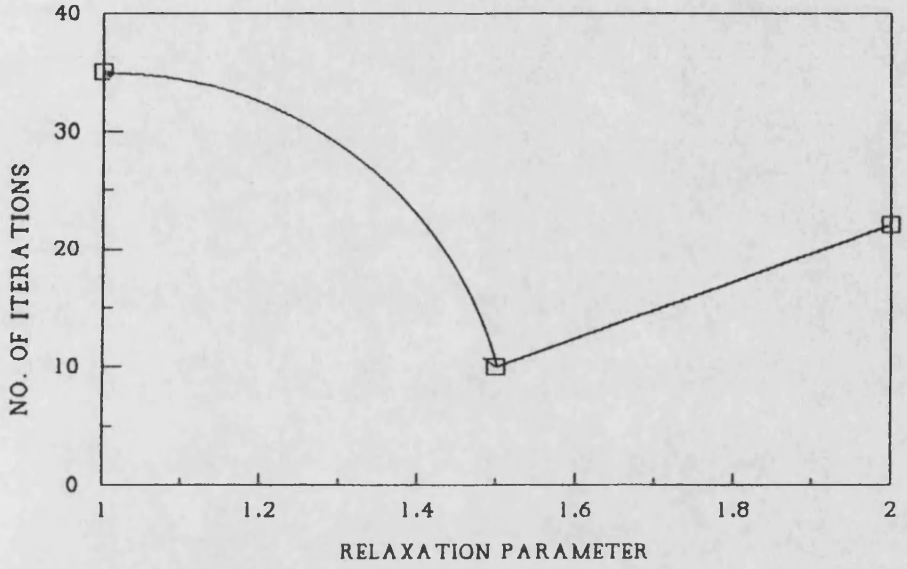

FIGURE B-4 PSOR AND LSOR RELAXATION PARAMETER

Generally, the closer the system to homogeneous state the closer $\omega_{opt.}$ to 1.0.

3- Alternating Direction Implicit (ADI) method

This method is introduced by D. W. Peaceman and H. H. Rachford as well as by J. Douglas. The method involves essentially one row, X-direction, iteration at a time solving for the unknown in this direction, followed by one column, Y-direction, iteration at a time solving for the unknown values.

X-direction iteration

The finite-difference equations are written for each point in the row. This procedure generate a simultaneous liner equations which are of a tridiagonal form. The general equation can be written as:

$$a_M U_W + b_{xM} U_M + c_M U_E = d_{xM} \qquad (B-7.1)$$

Where:

$$b_{xM} = b_M + e_M + f_M - \xi\tau \qquad (B-7.2)$$

$$d_{xM} = d_M - e_M U_N^k + (e_M + f_M - \xi\tau) U_M^k - f_M U_S^k \qquad (B-7.3)$$

τ = an acceleration parameter calculated by Equation (B-11)

ξ_M = the normalizing parameter calculated by Equation (B-13)

Equation (B-7.1) solved by Thomas algorithm and U values are obtained.

Y-direction iteration

The general equation for the iteration in Y-direction is:

$$e_M U_N^{k-1} + b_{yM} U_M^{k+1} + f_M U_S^{k+1} = d_{yM} \qquad (B-8.1)$$

Where byM and dyM are defined as:

$$b_{yM} = b_m + a_M + c_M - \xi_M \tau \qquad (B-8.2)$$

$$d_{yM} = d_M - a_M U_W + (a_M + c_M - \xi_M \tau) U_M - c_M U_E \qquad (B-8.3)$$

ADI Acceleration parameter

ADI Acceleration parameters, to speed-up the rate of convergence, is estimated by Peaceman-Rachford[1] method. The method utilized interblock transmissibility and the number of cell in the x- and y-direction (Nx, Ny) of the model to determine the upper and lower limits. These parameters are

$$M_1 = \frac{\pi^2}{(2NX^2)\left(1 + \left(\frac{\Delta x^2 k_y}{\Delta y^2 k_x}\right)\right)} \qquad (B-9.1)$$

$$M_2 = \frac{2}{1 + \left(\frac{\Delta x^2 k_y}{\Delta y^2 k_x}\right))} \qquad (B-9.2)$$

$$M_3 = \frac{\pi^2}{(2NY^2)\left(1 + \left(\frac{\Delta y^2 k_x}{\Delta x^2 k_y}\right)\right)} \qquad (B-9.3)$$

$$M_4 = \frac{2}{1 + \left(\frac{\Delta y^2 k_x}{\Delta x^2 k_y}\right))} \qquad (B-9.4)$$

The lower limit of the acceleration parameter is

$$\tau_{min} = \min(M_1, M_3) \qquad (B-9.5)$$

The upper limit of the acceleration parameter is

$$\tau_{max} = \max(M_2, M_4) \qquad (B-9.5)$$

Bjordammen and Coats[2] have suggested the following procedure for two-dimensional areal problems.

τ_{min} = minimum over the grid of

$$\left[\frac{\pi^2}{(2NX^2)\left(1+\left(\frac{\Delta x^2 k_x}{\Delta y^2 k_y}\right)\right)}, \frac{\pi^2}{(2NY^2)\left(1+\left(\frac{\Delta y^2 k_y}{\Delta x^2 k_x}\right)\right)}\right]$$

$$\tau_{max} = 1 \qquad (B-10)$$

Then the ADI acceleration parameters are distributed geometrically[3] [4] between the minimum and maximum values as :

$$\tau^{k+1} = \tau^k r \qquad (B-11)$$

Where 'r' is calculated as follows:

$$r = \left(\frac{\tau_{min}}{\tau_{max}}\right)^{\frac{1}{m-1}} \qquad (B-12)$$

m = number of acceleration parameters (4 or 6 are normally used)

ADI Normalizing Factor

Acceleration parameters must be normalized to account for variable grids size and directional permeability . The normalizing factor computed from :

$$\xi_M = \frac{(hk)_{avg.}}{\Delta X_M \Delta Y_M} \qquad (B-13.1)$$

$$hk_{avg.} = \frac{1}{N}\sum_{i=1}^{N}\sqrt{K_{xM} K_{yM}} \qquad (B-13.2)$$

1.1 References

1 - Peaceman, D. W. and Rachford, H. H.: "The Numerical Solution of Parabolic and Elliptic Differential Equation." J. Soc. Indust. Appl. Math., 3, (1955), 28-41.

2 - Bjordammen, J. B. and Coats, K. H.: "Comparison of Alternating Direction and Successive Overrelaxation Techniques in Simulation of Reservoir Fluid Flow." Soc. Petrol. Eng. J., 9, No. 2, (1969), 47-58.

3 - Birkhoff, G., Varga, R. S. and Yound, D.: "Alternating Direction Implicit Method." Advances in Computers, 3, (1962), 189-273.

4 - Brietenbach, E. A., Thurnau, and D. H. and Van Poolen, H. K.: "Solution of the Immiscible Fluid Flow Simulation Equations." Soc. Pet. Eng. J., (June 1969), 155-69.

APPENDIX C

INPUT DATA

APPENDIX C

INPUT DATA

TABLE C-1 GEOMETRY PROPERTIES				
PARAMERTER NAME	RUN:1	RUN:2	RUN:3	RUN:4
Block in x-direction	5	1	5	5
Block in y-direction	5	3	5	5
Block in z-direction	-	-	-	5
Reservoir thickness	100	40	40	40
Grid lenght in x, ft	100	100	200	100
Grid Length in y, ft	100	100	200	100

Rock Properteis[1, 2,]

TABLE C-2 ROCK PROPERTIES		
PARAMETER NAME	VALUE	CORRELATION
Initial Porosity, frac	0.20	
Absolute permeability, darcies	0.30	
Rock compressibility, psia^{-1}	1.0x10^{-4}	
Residual oil saturation	0.10	
Critical water saturation	0.20	
Critical gas saturation	0.00	
Oil relative permeability		Equation (C-1a)
Water relative permeability		Equation (C-1b)
Gas relative permeability		Equation (C-1c)
Water-Oil capillary pressure		Equation (C-2a)
Gas-Oil capillary pressure		Equation (C-2b)
Rock density, lb/ft^3	165.0	
Specific heat of the formation, Btu/lb-f	0.20	
Thermal conductivity of the formation, Btu/D-ft-f	18.5	
Thermal conductivity of overburden, Btu/D-ft-f	18.5	
Initail temperature, f	100.0	

Oil, water and gas relative permeabilities are considered temperature independent.The calculation are based upon the Naar-Henderson equations:

$$k_{ro} = \frac{S_o^3(1 - S_g + S_w - 2S_{wr})}{(1 - S_{wr})^4} \qquad (C-1a)$$

$$k_{rw} = \frac{(S_w - S_{wr})^4}{(1 - S_{wr})^4} \qquad (C-1b)$$

$$k_{rg} = \frac{S_g^3(2 - S_g - S_{wr})}{(1 - S_{wr})^4} \qquad (C-1c)$$

Capillary pressure relationship for oil-water is calculated by:

$$p_{cow} = (A_1 + B_1 S_w) \qquad if\ s_w \geq s_{wr}$$

$$= (A_2 + B_2 S_{wr}) \qquad if\ S_w < s_{wr} \qquad (C-2a)$$

Where the values of A and B in equation (C-2a) are:

A1 = 1.2045x10^{-1} B1 = 1.27127x10^{-1}

A2 = 8.127 B2 = 40.160

Capillary pressure relationship for oil-gas is represented by:

$$p_{cgw} = (A_1 + B_1 S_g) \qquad if\ s_g \geq s_{gr}$$

$$= (A_2 + B_2 S_{gr}) \qquad if\ S_g < s_{gr} \qquad (C-2b)$$

Where the values of A and B in the last equation are :

A1 = 1.11058x10^{-1} B1 = 2.2029x10^{-1}

A1 = 8.9670 B1 = 44.50

Equations (C-2) are presented graphically in the following figures:

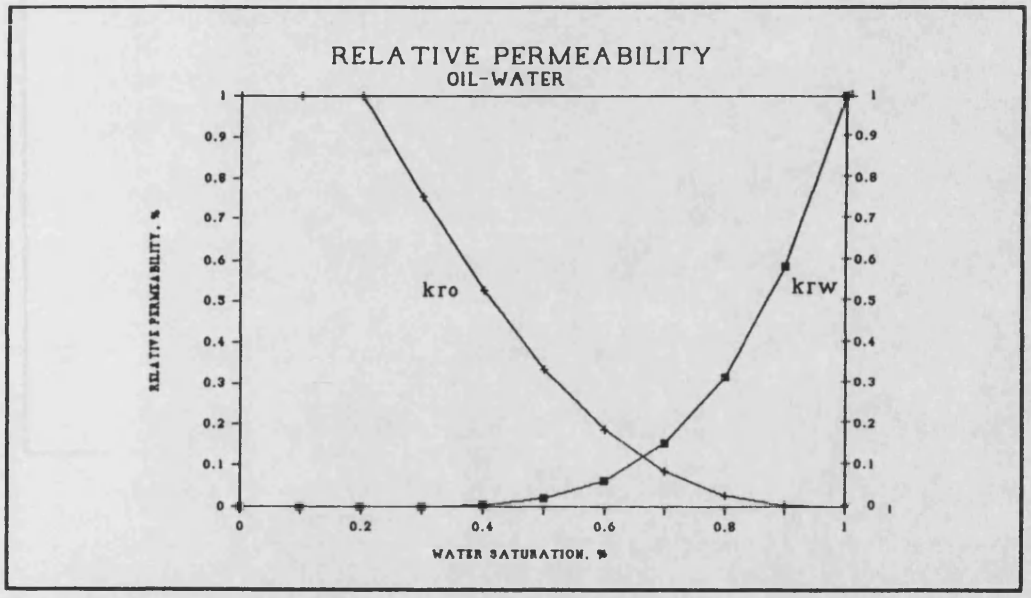

FIGURE C-1 RELATIVE PERMEABILITY OF OIL-WATER SYSTEM

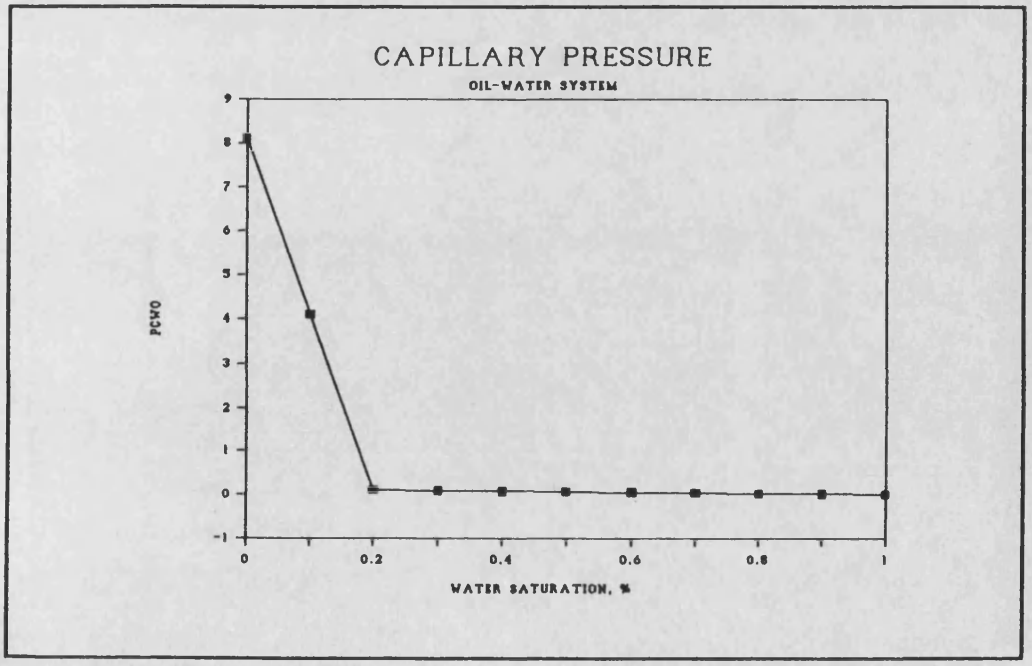

FIGURE C-2 CAPILLARY PRESSURE OF OIL-WATER SYSTEM

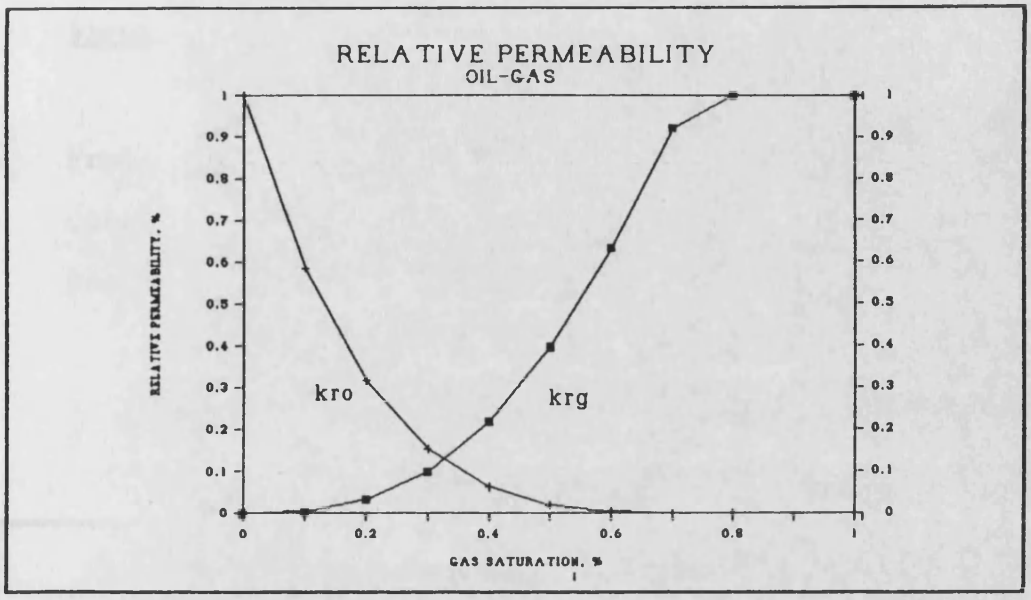

FIGURE C-3 RELATIVE PERMEABILITY OF OIL-GAS SYSTEM

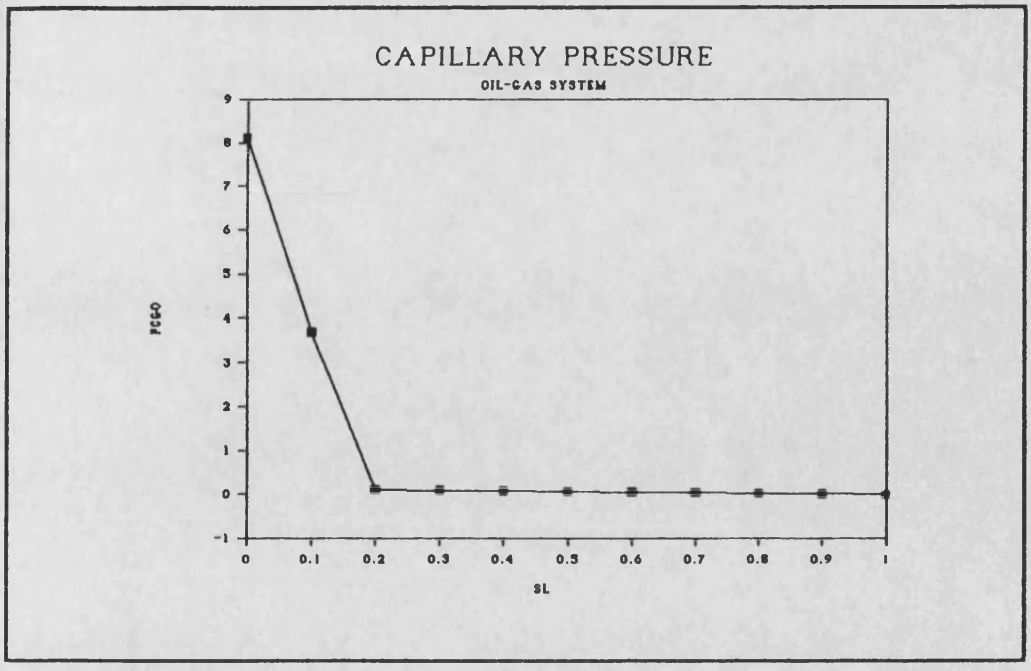

FIGURE C-4 CAPILLARY PRESSURE OF OIL-GAS SYSTEM

Fluid properties[3, 4]

Properties of reservoir fluids for runs 1, 2 and 3 are considered to be function of pressure only. In run 4 fluid properties are function of pressure and temperature.

A-Oil

TABLE C-3 OIL PROPERTIES		
PROPERTY NAME	VALUE	CORRELATION
Compressibility Co at Psat, psi^{-1}	0.30x10^{-5}	
Saturation pressure Psat, psi	2000.0	
Thermal expansion coeffi-cient, f^{-1}	0.40x10^{-3}	
Formation volume factor Bo, bbl/STB		Equation (B-.3a)
Solution gas-oil ratio Rso, SCF/BBl		Equation (B-.3b)
Density, lb/ft^3		Equation (B-.3c)
Viscosity, cp		Equation (B-.3d)

The oil formation volume factor at temperature T and pressure P can be estimated from the given formation volume factor at temperature Ti and pressure Pi by the following equation:

$$B_o = B_{oi}\left(1 + c_o(P - P_i) - \beta_o(T - T_i)\right) \qquad (C-3a)$$

Where in all the runs Pi is setup to be equal to saturation pressure (P_{sat}) and Ti equal reservoir initial temperature. The formation volume factor reachs the pick at P_{sat} after that begins to decline as shown in figure C-5.

Solution gas-oil ratio, at constant temperature, increases with pressure as shown in figure C-6. The Rso is expressed by the following equation:

$$RS_o = A_1 + (B_1 + C_1 P)P \qquad (C-3b)$$

Where the values of the constants are :

A1 = 82.137 B1 = 2.1568×10^{-1} C1 = -6.8143×10^{-6}

Oil Density

$$\rho_o = \frac{(A_2 + B_2 RRS_o)}{B_o} \qquad (C-4c)$$

Where the constants in the above equation are :

A2 = 54.670 B2 = 0.1173×10^{-1}

Viscosity of oil

$$\mu_o = A_3 + (B_3 + C_3 P)P \qquad (C-3d)$$

Where the values of the constants are :

$A_3 = 1.0060$ $B^3 = -5.8105 \times 10^{-4}$ $C_3 = 1.4046 \times 10^{-7}$

Oil properties in equations C-3 are plotted in the following figures:

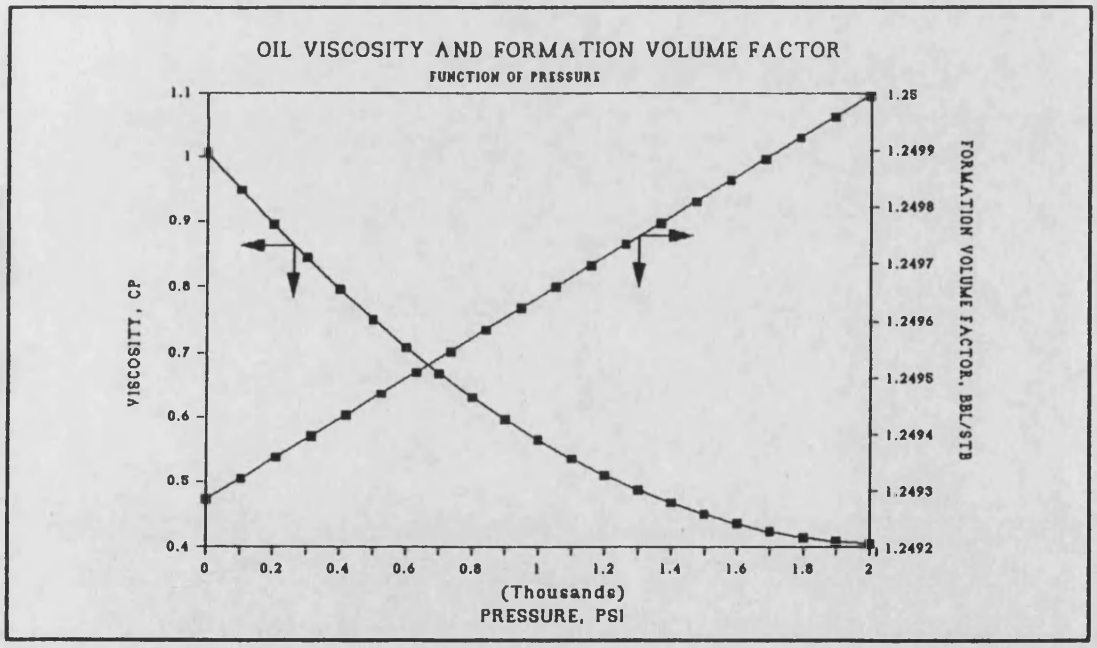

FIGURE C-5 OIL VISCOSITY AND FORMATION VOLUME FACTOR

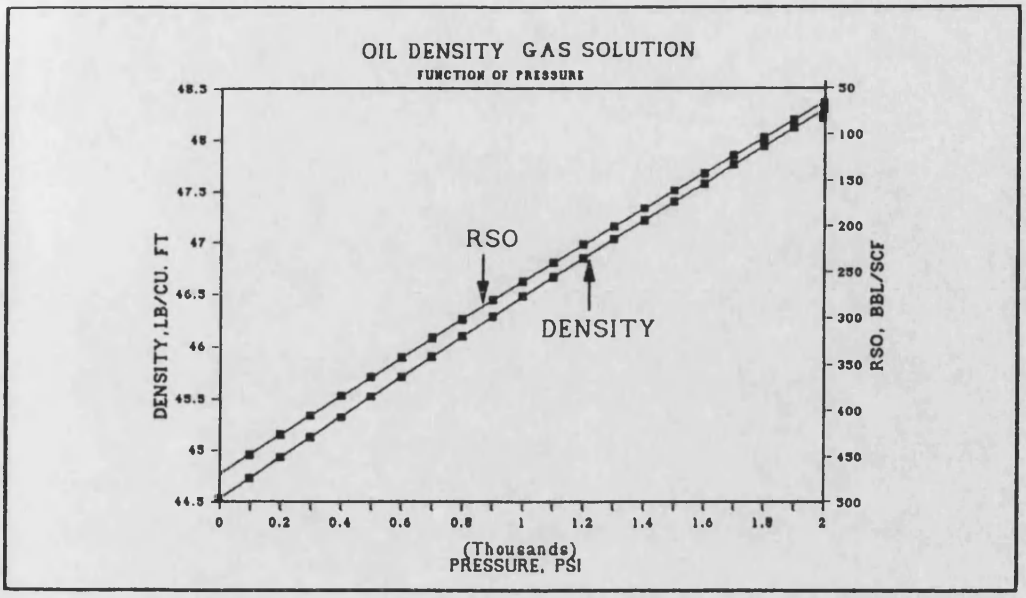

FIGURE C-6 OIL DENSITY AND GAS SOLUTION

B-Water

TABLE C-4 WATER PROPERTIES		
PROPERTY NAME	***VALUE***	***CORRELATION***
Compressibility at Psat, psi^{-1}	0.30x10^{-5}	
Thermal expansion coefficient, f^{-1}	0.40x10^{-3}	
Formation volume factor, bbl/stb		Equation (C-4a)
Density, lb/ft^3		Equation (C-4b)
Viscosity, cp	1.0	

$$B_w = A_1 + B_1 P \qquad (C-4a)$$

Where the values of the constants in the above equation are :

A1 = 1.00534 B1 = -2.8674x10-6

$$RS_w = A_2 + B_2 P \qquad (C-4b)$$

Where the constants are :

A2 = 1.05 B2 = 5.0x10-1

$$\rho_w = \frac{(A_3 + B_3 RS_w)}{B_w} \qquad (C-4c)$$

Where the values of the constants in the above equations are :

A3 = 64.4 B3 = 1.173x10-2

Water properties in equations C-4 are plotted in the following figures:

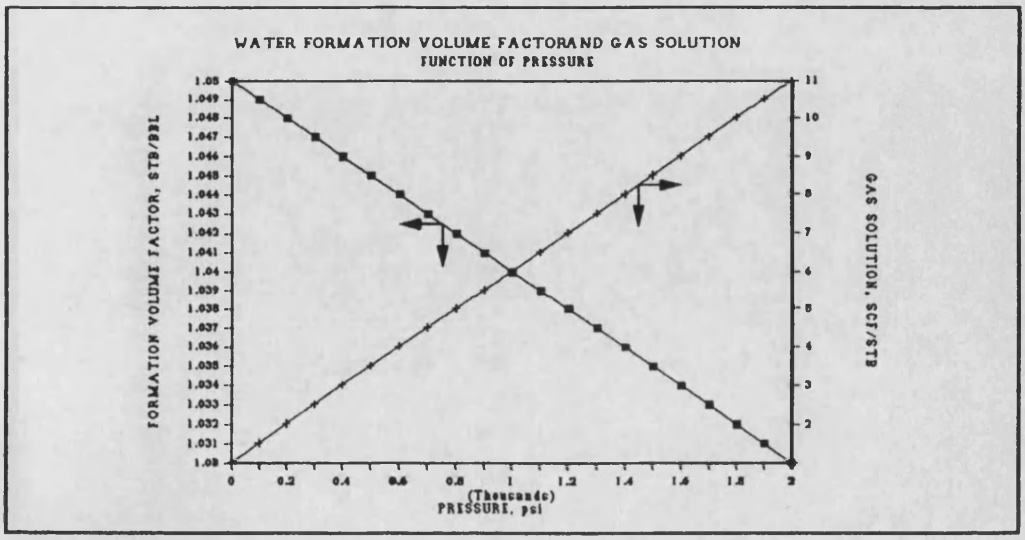

FIGURE C-7 WATER VISCOSITY AND FORMATION VOLUME FACTOR

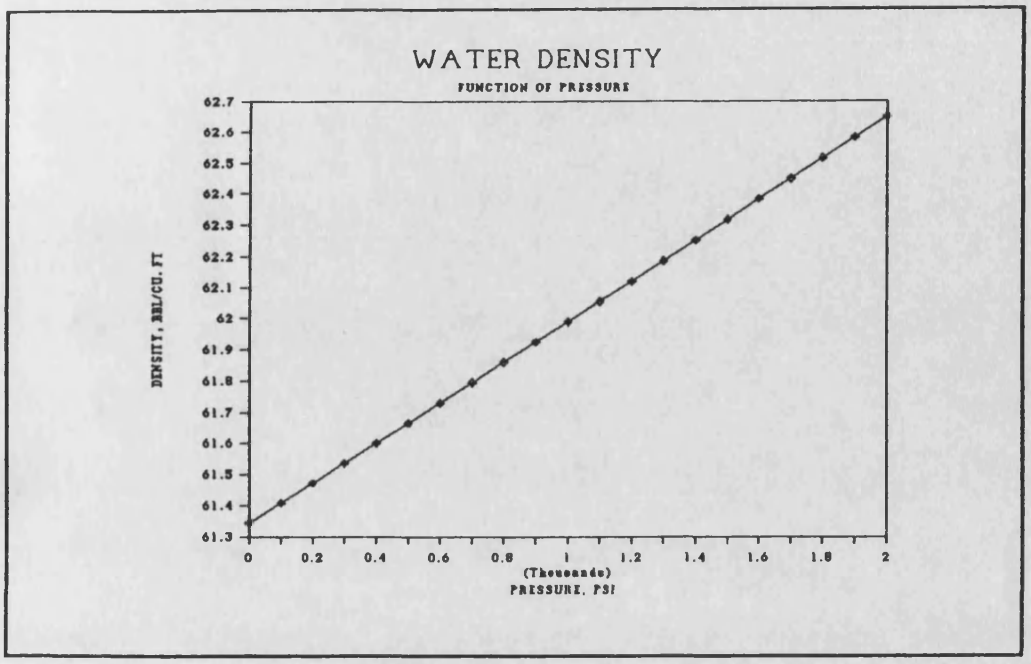

FIGURE C-8 WATER DENSITY

C-Gas

TABLE C-5 GAS PROPERTIES		
PROPERTY NAME	VALUE	CORRELATION
Compressibility at P, psi	0.30×10^{-5}	
Formation volume factor, bbl/stb		Equation (C-5a)
Density, lb/ft^3		Equation (C-5b)
Viscosity, cp		Equation (C-5c)

Gas formation volume factor is calculated as:

$$B_g = A_1 + \frac{B_1}{P} \qquad (C-5a)$$

$A_1 = -7.50 \times 10^{-5}$ $B_1 = 6.9458$

Density of gas

$$\rho_g = \frac{(A_2 X B_2)}{B_g} \qquad (C-5b)$$

$A_2 = 4.6921 \times 10^{-4}$ $B^2 = 25.0$

Gas viscosity

$$\mu_g = A_1 + (B_1 + (C_1 + D_1 P)P)P \qquad (C-5c)$$

Where the values of the constants in the above equations are :

$A_3 = 1.0864 \times 10^{-2}$ $B_3 = 2.5696 \times 10^{-6}$ $C_3 = -1.0207 \times 10^{-9}$ $D_3 = 2.7471 \times 10^{-13}$

Equations C-5 are presented graphically in the following figures:

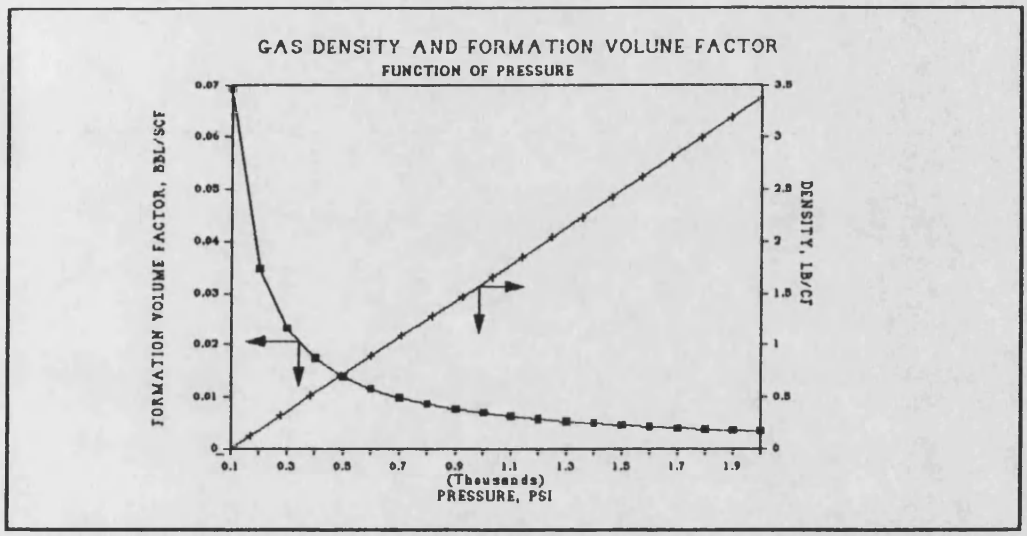

FIGURE C-9 GAS DENSITY AND FORMATION VOLUME FACTOR

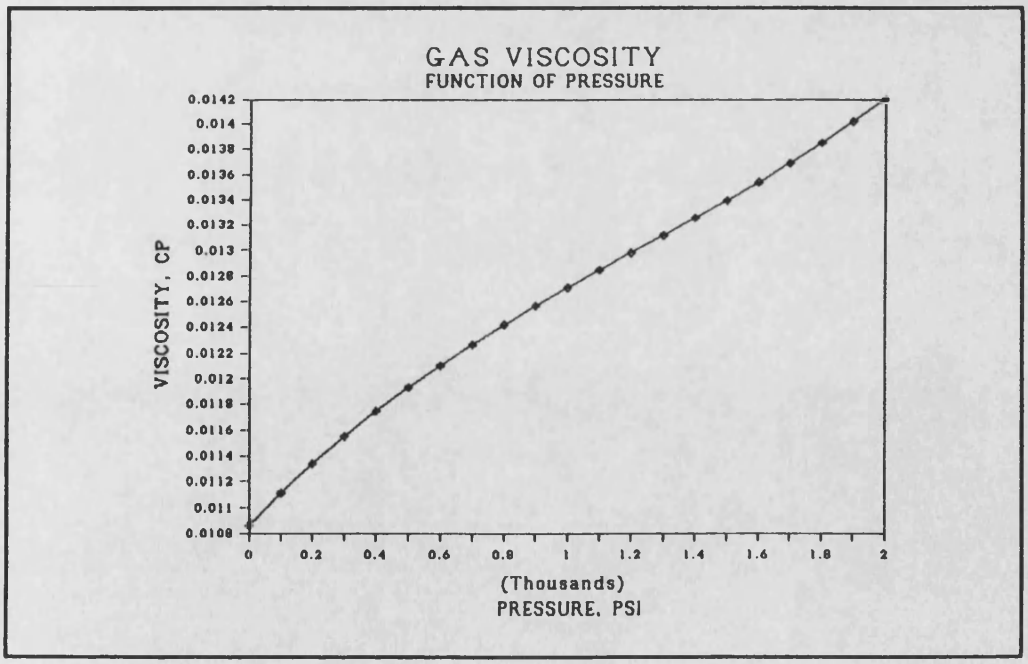

FIGURE C-10 GAS VISCOSITY

Oil thermal properties [5, 6, 7]

The oil density at temperature T and pressure P can be estimated from the given density at temperature Ti and pressure Pi by the following equation:

$$\rho_o = \rho_{ot}\left(1 + c_o(P - P_i) - \beta_o(T - T_i)\right) \qquad (C-6a)$$

Variation of Oil viscosity with temperature is based upon Andrade's equation.

$$\mu_o = A_1 e^{B_1/(T+460)} \qquad (C-6b)$$

Where the values of the constants are :

A1 = 1.882x10-4 B1 = 5000

The oil specific heat of oil is determined by:

\eqn "C sub po = A sub 2 + B sub 2 (T - 80)

Where the values of the constants are :

A2 = .455 B2 = 6.0x10-4

Water thermal properties: [5 6 7]

Water density is calculated as:

$$\rho_w = \rho_{wt}\left(1 + c_w(P - P_i) - \beta_w(T - T_i)\right) \qquad (C-7a)$$

The viscosity of water is essential paramerter in the thermal calculation. Gottfried's equation is used to

estimate water viscosity.

$$\mu_w = \frac{(A_1 - T)}{(B_1 T - C_1)} \qquad (C-7b)$$

Where the values of the constants in the last equations are :

A1 = 1776 B1 = 26.5 C1 = 89

Gas thermal properties:[5] [6] [7]

Gas viscosity is estimated as:

$$\mu_g = A_1 + B_1 T \qquad (C-8a)$$

Where the values of the constants in the equations are :

A1 = 8.197×10^{-3} B1 = 2.015×10^{-5}

Gas density:

$$\rho_g = \frac{(A_2 P)}{(T + 460)} \qquad (C-8b)$$

Where the values of the constants in the equations are :

A2 = 2.7

C. References

1 - "Fundamentals of Special Core Analysis" Core Laboratories INC.

2 - Van Poolen, H. K: "Lecture note on Reservoir Engineering: Applied." Denver, (Aug. 1982)

3 - "A Course in the Phase Behavior of Hydrocarbon Reservoir Fluid." Core Laboratories, INC.

4 - Crichlow, H. B.: "Modern Reservoir Engineering - A simulation Approach." Prentice-Hall Inc. (1977)

5 - Ferrer, J. and Farouq Ali, S. M.: "A three-phase, Two-Dimensional Compositional Thermal Simulator for Steam Injection Processes." J. Cand. Pet. Tech. (Jan-March 1977).

6 - "Computer Simulation of Thermal Enhanced Oil Recovery Schemes." Computer Modelling Group(Canada), and Oil Recovery Division(England), Dorchester, Dorset, England (Jun. 8-10, 1982)

7 - AbdAlah A. .: " A Three-Phase, Two Dimensional Numerical Simulation of The Steam Flood Process." Phd, Univ. of Texas, (1970)

APPENDIX D

SAMPLE OUTPUT

APPENDIX D
SAMPLE OUTPUT

```
*********************************************************
*                                                       *
*                                                       *
*                                                       *
*   SIMULATION TEST ( STEAM.2D200X100 )                 *
*                                                       *
*                                                       *
*                                                       *
*                                                       *
*       PLOT OPTION    :  OFF                           *
*                                                       *
*       PRINT OPTION   :   ON                           *
*                                                       *
*       CHECK OPTION   :  OFF                           *
*                                                       *
*        HEAT OPTION   :  OFF                           *
*                                                       *
*       UNITE OPTION   : ENGLISH UNITE                  *
*                                                       *
*                                                       *
*                                                       *
*                              RUN NO. = ........       *
*                                                       *
*********************************************************
```

```
     NUMBER OF ROWS      =   5
   NUMBER OF COLUMNS     =   5

      j =      1          2          3          4          5
DELTA - x     100.0      100.0      100.0      100.0      100.0


 DELTA - y
  i =  1        200.0
  i =  2        200.0
  i =  3        200.0
  i =  4        200.0
  i =  5        200.0


    RESERVOIR LAY_OUT
         VALUE = 0 FOR BLOCK outside the reserior
         value = 1 for block inside the reservoir
```

*** GRID INDEX ****

	j= 1	2	3	4	5
I= 1	1	1	1	1	1
I= 2	1	1	1	1	1
I= 3	1	1	1	1	1
I= 4	1	1	1	1	1
I= 5	1	1	1	1	1

*** POROSITY TABLE ****

	j= 1	2	3	4	5
I= 1	0.2000	0.2000	0.2000	0.2000	0.2000
I= 2	0.2000	0.2000	0.2000	0.2000	0.2000
I= 3	0.2000	0.2000	0.2000	0.2000	0.2000
I= 4	0.2000	0.2000	0.2000	0.2000	0.2000
I= 5	0.2000	0.2000	0.2000	0.2000	0.2000

*** THICKNESS TABLE ****

	j= 1	2	3	4	5
I= 1	40.0000	40.0000	40.0000	40.0000	40.0000
I= 2	40.0000	40.0000	40.0000	40.0000	40.0000
I= 3	40.0000	40.0000	40.0000	40.0000	40.0000
I= 4	40.0000	40.0000	40.0000	40.0000	40.0000
I= 5	40.0000	40.0000	40.0000	40.0000	40.0000

*** DEPTH-SUB_SEA TABLE ****

	j= 1	2	3	4	5
I= 1	1000.00001000.0000	1000.0000	1000.0000	1000.000	
I= 2	1000.00001000.0000	1000.0000	1000.0000	1000.0000	
I= 3	1000.00001000.0000	1000.0000	1000.0000	1000.0000	
I= 4	1000.00001000.0000	1000.0000	1000.0000	1000.0000	
I= 5	1000.00001000.0000	1000.0000	1000.0000	1000.000	

*** X-DIRECTION PERMEABILITY ****

	j= 1	2	3	4	5
I= 1	0.3000	0.3000	0.3000	0.3000	0.3000
I= 2	0.3000	0.3000	0.3000	0.3000	0.3000
I= 3	0.3000	0.3000	0.3000	0.3000	0.3000
I= 4	0.3000	0.3000	0.3000	0.3000	0.3000
I= 5	0.3000	0.3000	0.3000	0.3000	0.3000

Y-DIRECTION PERMEABILITY *

	j= 1	2	3	4	5
I= 1	0.3000	0.3000	0.3000	0.3000	0.3000
I= 2	0.3000	0.3000	0.3000	0.3000	0.3000
I= 3	0.3000	0.3000	0.3000	0.3000	0.3000
I= 4	0.3000	0.3000	0.3000	0.3000	0.3000
I= 5	0.3000	0.3000	0.3000	0.3000	0.3000

*** TRANSMISSIBILITY B_CONSTANT ****

j=	1	2	3	4	5
I= 1	27.0480	27.0480	27.0480	27.0480	27.0480
I= 2	27.0480	27.0480	27.0480	27.0480	27.0480
I= 3	27.0480	27.0480	27.0480	27.0480	27.0480
I= 4	27.0480	27.0480	27.0480	27.0480	27.0480
I= 5	0.0000	0.0000	0.0000	0.0000	0.0000

*** TRANSMISSIBILITY D_CONSTANT ****

j=	1	2	3	4	5
I= 1	0.0000	6.7620	6.7620	6.7620	6.7620
I= 2	0.0000	6.7620	6.7620	6.7620	6.7620
I= 3	0.0000	6.7620	6.7620	6.7620	6.7620
I= 4	0.0000	6.7620	6.7620	6.7620	6.7620
I= 5	0.0000	6.7620	6.7620	6.7620	6.7620

*** TRANSMISSIBILITY F_CONSTANT ****

j=	1	2	3	4	5
I= 1	6.7620	6.7620	6.7620	6.7620	0.0000
I= 2	6.7620	6.7620	6.7620	6.7620	0.0000
I= 3	6.7620	6.7620	6.7620	6.7620	0.0000
I= 4	6.7620	6.7620	6.7620	6.7620	0.0000
I= 5	6.7620	6.7620	6.7620	6.7620	0.0000

**** TRANSMISSIBILITY H_CONSTANT ****

	j= 1	2	3	4	5
I= 1	0.0000	0.0000	0.0000	0.0000	0.0000
I= 2	27.0480	27.0480	27.0480	27.0480	27.0480
I= 3	27.0480	27.0480	27.0480	27.0480	27.0480
I= 4	27.0480	27.0480	27.0480	27.0480	27.0480
I= 5	27.0480	27.0480	27.0480	27.0480	27.0480

*** ROCK BULK VOLUEM ****

	J= 1	2	3	4	5
I= 1	0.800D+06	0.800D+06	0.800D+06	0.800D+06	0.800D+06
I= 2	0.800D+06	0.800D+06	0.800D+06	0.800D+06	0.800D+06
I= 3	0.800D+06	0.800D+06	0.800D+06	0.800D+06	0.800D+06
I= 4	0.800D+06	0.800D+06	0.800D+06	0.800D+06	0.800D+06
I= 5	0.800D+06	0.800D+06	0.800D+06	0.800D+06	0.800D+06

TEMPERATURE DISTRIBUTION *

	j= 1	2	3	4	5
I= 1	100.0000	100.0000	100.0000	100.0000	100.0000
I= 2	100.0000	100.0000	100.0000	100.0000	100.0000
I= 3	100.0000	100.0000	100.0000	100.0000	100.0000
I= 4	100.0000	100.0000	100.0000	100.0000	100.0000
I= 5	100.0000	100.0000	100.0000	100.0000	100.0000

*** INITIAL OIL PRESSURE ****

	j= 1	2	3	4	5
I= 1	1900.0000	1900.0000	1900.0000	1900.0000	1900.000
I= 2	1900.0000	1900.0000	1900.0000	1900.0000	1900.0000
I= 3	1900.0000	1900.0000	1900.0000	1900.0000	1900.0000

```
                                  ^^^^^^^
I= 4 1900.00001900.0000  1900.0000  1900.0000      1900.0000

I= 5 1900.00001900.0000  1900.0000  1900.0000      1900.000
       *******                                     *******
```

INITIAL WATER PRESSURE *

```
   j=     1           2           3           4           5

I= 1 1899.93041899.9304  1899.9304  1899.9304      1899.930
       *******                                     *******
I= 2 1899.93041899.9304  1899.9304  1899.9304      1899.9304

I= 3 1899.93041899.9304  1899.9304  1899.9304      1899.9304
                                 ^^^^^^^

I= 4 1899.93041899.9304  1899.9304  1899.9304      1899.9304

I= 5 1899.93041899.9304  1899.9304  1899.9304      1899.930
       *******                                     *******
```

*** INITIAL OIL SATURATION ****

```
   j=     1           2           3           4           5

I= 1    0.5500    0.5500      0.5500      0.5500      0.5500
       *******                                     *******
I= 2    0.5500    0.5500      0.5500      0.5500      0.5500

I= 3    0.5500    0.5500      0.5500      0.5500      0.5500
                                ^^^^^^^

I= 4    0.5500    0.5500      0.5500      0.5500      0.5500

I= 5    0.5500    0.5500      0.5500      0.5500      0.5500
       *******                                     *******
```

*** INITIAL WATER SATURATION ****

```
   j=     1           2           3           4           5

I= 1    0.4000    0.4000      0.4000      0.4000      0.4000
       *******                                     *******
I= 2    0.4000    0.4000      0.4000      0.4000      0.4000

I= 3    0.4000    0.4000      0.4000      0.4000      0.4000
                                ^^^^^^^

I= 4    0.4000    0.4000      0.4000      0.4000      0.4000

I= 5    0.4000    0.4000      0.4000      0.4000      0.4000
```

******* *******

*** INITIAL GAS SATURATION ****

j=	1	2	3	4	5
I= 1	0.0500	0.0500	0.0500	0.0500	0.0500
	*******				*******
I= 2	0.0500	0.0500	0.0500	0.0500	0.0500
I= 3	0.0500	0.0500	0.0500	0.0500	0.0500
			^^^^^^^		
I= 4	0.0500	0.0500	0.0500	0.0500	0.0500
I= 5	0.0500	0.0500	0.0500	0.0500	0.0500
	*******				*******

TIME STEP DELTA = 5.0
MINIMUM TIME STEP = 1.0
MAXIMUM TIME STEP = 0.0
TOTAL SIMULATION TIME = 40.0
PRINT OUT INTERVAL = 0.0
CALANDER YEAR = 1985.

ADIP TOLERANCE = 0.0100 psi
EXTRAPOLATED SATURATION TOLERANCE = 0.0500
EXTRABOLATED PRESSURE TOLERANCE = ****** psi
MATERIAL BALANCE TOLERANCE = 0.1000

THETA = 1.000
GRAVITY MULTIPLIER = 0.006940
EXTRAPOLATION FACTOR = 1.000
WELL PORE RADIUS = 0.500

MAXIMUM OIL SATURATION (somax) = 0.9000
MINIMUM OIL SATURATION (sor) = 0.1000
MAXIMUM GAS SATURATION (sgmax) = 0.8000
MINIMUM GAS SATURATION (sgr) = 0.0000
MINIMUM WATER SATURATION (swr) = 0.1000

NUMBER OF ITERATION PARAMETER FOR ADI = 6
ALPHA MAXIMUM = 0.9605
THE ITERATION PARAMETERS ARE :

ALPHA(1) = 0.0000
ALPHA(2) = 0.4761
ALPHA(3) = 0.7255
ALPHA(4) = 0.8562
ALPHA(5) = 0.9246

ALPHA(6) = 0.9605

INITIAL OIL IN PLACE = 0.301661D+06 STB
INITIAL WATER IN PLACE = 0.276350D+06 STB
INITIAL GAS IN PLACE = 0.153823D+09 SCF

**** WELL DATA ****

NUMBER OF WELLS = 5

THE INTERNALLY GENERATED FLOW MATRIX IS :

ITYPE = INDEX FOR SPECIFIED THE WELL TYPE . IF
= 1 OIL INJECTION WELL WITH (+) FLOW RATE
= 2 WATER INJECTION WELL WITH (+) FLOW RATE
= 3 GAS INJECTION WELL WITH (+) FLOW RATE
= 4 OIL PRODUCING WELL WITH (-) FLOW RATE
= 5 WATER PRODUCING WELL WITH (-) FLOW RATE
= 6 GAS PRODUCING WELL WITH (-) FLOW RATE
= 7 TOTAL PRODUCTION OF THE FLUID IS SPECIFIED
= 8 CONSTANT OIL PRESSURE IS SPECIFIED
= 9 HEAT INJECTION

RAW I	COLUMN J	ITYPE	FLOW RATE
1	1	4	-50.0
5	1	4	-50.0
1	5	4	-50.0
5	5	4	-50.0
3	3	2	200.0

INTERNALLY GENERATED INDICATOR ARRAY :

IND = 0	FOR BLOCK THAT OUTSIDE GRID
IND = 1	FOR BLOCK INSIDE GRID THAT IS NOT A WELL
IND < 0	FOR BLOCK INSIDE GRID THAT IS A WELL
IND = + 5	FOR BLOCK WITH ERROR

*** GRID INDEX DISTRIBUTIO ****

	j= 1	2	3	4	5
I= 1	-1	1	1	1	-3
I= 2	1	1	1	1	1
I= 3	1	1	-5	1	1
I= 4	1	1	1	1	1
I= 5	-2	1	1	1	-4

PRODUCTION DATA AFTER 1 TIME STEPS
TIME = 5.00 DAYS

*** OIL PRESSURE ****

	j= 1	2	3	4	5
I= 1	1894.2278	1898.8570	1901.4065	1898.8570	1894.227
	*******				*******
I= 2	1895.9481	1899.3409	1902.7091	1899.3409	1895.9481
I= 3	1896.5012	1899.7707	1905.8953	1899.7707	1896.5012
			^^^^^^^		
I= 4	1895.9481	1899.3409	1902.7091	1899.3409	1895.9481
I= 5	1894.2278	1898.8570	1901.4065	1898.8570	1894.22
	*******				*******

*** WATER PRESSURE ****

	j= 1	2	3	4	5
I= 1	1894.1582	1898.7874	1901.3369	1898.7874	1894.158
	*******				*******
I= 2	1895.8785	1899.2713	1902.6395	1899.2713	1895.8785
I= 3	1896.4316	1899.7011	1905.8302	1899.7011	1896.4316
			^^^^^^^		
I= 4	1895.8785	1899.2713	1902.6395	1899.2713	1895.8785
I= 5	1894.1582	1898.7874	1901.3369	1898.7874	1894.158
	*******				*******

*** GAS PRESSURE ****

	j= 1	2	3	4	5
I= 1	1894.2181	1898.8470	1901.3964	1898.8470	1894.218
	*******				*******
I= 2	1895.9382	1899.3308	1902.6988	1899.3308	1895.9382
I= 3	1896.4914	1899.7607	1905.8922	1899.7607	1896.4914
			^^^^^^^		
I= 4	1895.9382	1899.3308	1902.6988	1899.3308	1895.9382
I= 5	1894.2181	1898.8470	1901.3964	1898.8470	1894.218
	*******				*******

*** OIL SATURATION ****

j=	1	2	3	4	5
I= 1	0.5482	0.5498	0.5501	0.5498	0.5482
	*******				*******
I= 2	0.5488	0.5498	0.5508	0.5498	0.5488
I= 3	0.548 9	0.5495	0.5183	0.5495	0.5489
			^^^^^^^		
I= 4	0.548 8	0.5498	0.5508	0.5498	0.5488
I= 5	0.5482	0.5498	0.5501	0.5498	0.5482
	*******				*******

*** WATER SATURATION ****

j=	1	2	3	4	5
I= 1	0.4000	0.4000	0.4000	0.4000	0.4000
	*******				*******
I= 2	0.4000	0.4000	0.4000	0.4000	0.4000
I= 3	0.4000	0.4000	0.4356	0.4000	0.4000
			^^^^^^^		
I= 4	0.4000	0.4000	0.4000	0.4000	0.4000
I= 5	0.4000	0.4000	0.4000	0.4000	0.4000
	*******				*******

*** GAS SATURATION ****

j=	1	2	3	4	5
I= 1	0.0518	0.0502	0.0499	0.0502	0.0518
	*******				*******
I= 2	0.0512	0.0502	0.0492	0.0502	0.0512
I= 3	0.0511	0.0505	0.0461	0.0505	0.0511
			^^^^^^^		
I= 4	0.0512	0.0502	0.0492	0.0502	0.0512
I= 5	0.0518	0.0502	0.0499	0.0502	0.0518
	*******				*******

**** WELL REPORT ****

ROW I	COLUMN J	OIL RATE (STB/D)	WATER RATE (STB/D)	GAS RATE (SCF/D)	GOR (SCF/STB)
1	1	-50.0	-1.1	-608.2	12.1632
5	1	-50.0	-1.1	-608.2	12.1632
1	5	-50.0	-1.1	-608.2	12.1632
5	5	-50.0	-1.1	-608.2	12.1632
3	3	0.0	200.0	0.0	0.0000

OILOLD = 0.301661D+06
WATOLD = 0.276350D+06
GASOLD = 0.153823D+09

OILNEW = 0.300661D+06
WATNEW = 0.277328D+06
GASNEW = 0.153353D+09

+ FOR FLUID INJECTION
- FOR FLUID PRODUCTION
OIL FLOW = -1000.0
WATER FLOW = 978.2
GAS FLOW =-0.468096D+06

OIL MATERIAL BALANCE =1.0000
WATER MATERIAL BALANCE =1.0001
GAS MATERIAL BALANCE =1.0042

NO. OF ADIP ITERATIONS = 20

NO. OF SATURATION LOOPS = 1
NO. OF PRESSURE LOOPS = 1
NO. OF MATERIAL BALANCE ITERATIONS = 1

PRESSURE AVERAGE =1900.0
OIL SATURATION AVERAGE =0.5481
WATER SATURATION AVERAGE =0.4014
GAS SATURATION AVERAGE =0.0505
TEMPERATURE AVERAGE =100.00
OIL RECOVERY IN THIS TIME STEP =0.3315

EXECUTION TIME (cpu) = 16.080429 SEC.

PRODUCTION DATA AFTER 2 TIME STEPS
TIME = 10.00 DAYS

*** OIL PRESSURE ****

	j= 1	2	3	4	5
I= 1	1891.8939	1897.4123	1900.5608	1897.4123	1891.893
	*******				*******
I= 2	1893.7287	1897.9025	1902.0104	1897.9025	1893.7287
I= 3	1894.3233	1898.2856	1906.5213	1898.2856	1894.3233
			^^^^^^^		
I= 4	1893.7287	1897.9025	1902.0104	1897.9025	1893.7287
I= 5	1891.8939	1897.4123	1900.5608	1897.4123	1891.893
	*******				*******

*** WATER PRESSURE ****

	j= 1	2	3	4	5
I= 1	1891.8242	1897.3427	1900.4912	1897.3427	1891.824
	*******				*******
I= 2	1893.6591	1897.8329	1901.9408	1897.8329	1893.6591
I= 3	1894.2537	1898.2160	1906.4606	1898.2160	1894.2537
			^^^^^^^		
I= 4	1893.6591	1897.8329	1901.9408	1897.8329	1893.6591
I= 5	1891.8242	1897.3427	1900.4912	1897.3427	1891.824
	*******				*******

*** GAS PRESSURE ****

	j= 1	2	3	4	5
I= 1	1891.8843	1897.4023	1900.5508	1897.4023	1891.884
	*******				*******
I= 2	1893.7190	1897.8925	1902.0003	1897.8925	1893.7190
I= 3	1894.3136	1898.2757	1906.5255	1898.2757	1894.3136
			^^^^^^^		
I= 4	1893.7190	1897.8925	1902.0003	1897.8925	1893.7190
I= 5	1891.8843	1897.4023	1900.5508	1897.4023	1891.884
	*******				*******

*** OIL SATURATION ****

j=	1	2	3	4	5
I= 1	0.5475	0.5495	0.5497	0.5495	0.5475
	*******				*******
I= 2	0.5481	0.5494	0.5501	0.5494	0.5481
I= 3	0.5482	0.5487	0.4854	0.5487	0.5482
			^^^^^^^		
I= 4	0.5481	0.5494	0.5501	0.5494	0.5481
I= 5	0.5475	0.5495	0.5497	0.5495	0.5475
	*******				*******

*** WATER SATURATION ****

j=	1	2	3	4	5
I= 1	0.4000	0.4000	0.4000	0.4000	0.4000
	*******				*******
I= 2	0.4000	0.4000	0.4004	0.4000	0.4000
I= 3	0.4000	0.4002	0.4700	0.4002	0.4000
			^^^^^^^		
I= 4	0.4000	0.4000	0.4004	0.4000	0.4000
I= 5	0.4000	0.4000	0.4000	0.4000	0.4000
	*******				*******

*** GAS SATURATION ****

j=	1	2	3	4	5
I= 1	0.0525	0.0505	0.0503	0.0505	0.0525
	*******				*******
I= 2	0.0519	0.0506	0.0495	0.0506	0.0519
I= 3	0.0518	0.0511	0.0447	0.0511	0.0518
			^^^^^^^		
I= 4	0.0519	0.0506	0.0495	0.0506	0.0519
I= 5	0.0525	0.0505	0.0503	0.0505	0.0525
	*******				*******

**** WELL REPORT ****

ROW I	COLUMN J	OIL RATE (STB/D)	WATER RATE (STB/D)	GAS RATE (SCF/D)	GOR (SCF/STB)
1	1	-50.0	-1.1	-766.1	15.3212
5	1	-50.0	-1.1	-766.1	15.3212
1	5	-50.0	-1.1	-766.1	15.3212
5	5	-50.0	-1.1	-766.1	15.3212
3	3	0.0	200.0	0.0	0.0000

OILOLD = 0.300661D+06
WATOLD = 0.277328D+06
GASOLD = 0.153353D+09

OILNEW = 0.299661D+06
WATNEW = 0.278306D+06
GASNEW = 0.152883D+09

+ FOR FLUID INJECTION
- FOR FLUID PRODUCTIO
OIL FLOW = -1000.0
WATER FLOW = 977.7
GAS FLOW =-0.470813D+06

OIL MATERIAL BALANCE =1.0000
WATER MATERIAL BALANCE =0.9999
GAS MATERIAL BALANCE =0.9969

NO. OF ADIP ITERATIONS = 17

NO. OF SATURATION LOOPS = 1
NO. OF PRESSURE LOOPS = 1
NO. OF MATERIAL BALANCE ITERATIONS = 1

PRESSURE AVERAGE =1896.8
OIL SATURATION AVERAGE =0.5463
WATER SATURATION AVERAGE =0.4028
GAS SATURATION AVERAGE =0.0509
TEMPERATURE AVERAGE =100.00
OIL RECOVERY IN THIS TIME STEP =0.3315

EXECUTION TIME (cpu) = 12.250022 SEC.

PRODUCTION DATA AFTER 3 TIME STEPS
TIME = 15.00 DAYS

*** OIL PRESSURE ****

	j= 1	2	3	4	5
I= 1	1890.1076	1895.8199	1899.1664	1895.8199	1890.107
	*******				*******
I= 2	1891.9500	1896.2999	1900.6595	1896.2999	1891.9500
I= 3	1892.5523	1896.6585	1905.9834	1896.6585	1892.5523
			^^^^^^^		
I= 4	1891.9500	1896.2999	1900.6595	1896.2999	1891.9500
I= 5	1890.1076	1895.8199	1899.1664	1895.8199	1890.107
	*******				*******

*** WATER PRESSURE ****

	j= 1	2	3	4	5
I= 1	1890.0380	1895.7503	1899.0968	1895.7503	1890.038
	*******				*******
I= 2	1891.8804	1896.2303	1900.5901	1896.2303	1891.8804
I= 3	1892.4827	1896.5890	1905.9269	1896.5889	1892.4827
			^^^^^^^		
I= 4	1891.8804	1896.2303	1900.5901	1896.2303	1891.8804
I= 5	1890.0380	1895.7503	1899.0968	1895.7503	1890.038
	*******				*******

*** GAS PRESSURE ****

	j= 1	2	3	4	5
I= 1	1890.0981	1895.8100	1899.1565	1895.8100	1890.098
	*******				*******
I= 2	1891.9405	1896.2901	1900.6497	1896.2901	1891.9405
I= 3	1892.5427	1896.6489	1905.9945	1896.6489	1892.5427
			^^^^^^^		
I= 4	1891.9405	1896.2901	1900.6497	1896.2901	1891.9405
I= 5	1890.0981	1895.8100	1899.1565	1895.8100	1890.098
	*******				*******

*** OIL SATURATION ****

	j= 1	2	3	4	5
I= 1	0.5469	0.5490	0.5491	0.5490	0.5469
	*******				*******
I= 2	0.5476	0.5489	0.5486	0.5489	0.5476
I= 3	0.5476	0.5476	0.4537	0.5476	0.5476
			^^^^^^^		
I= 4	0.5476	0.5489	0.5486	0.5489	0.5476
I= 5	0.5469	0.5490	0.5491	0.5490	0.5469
	*******				*******

*** WATER SATURATION ****

	j= 1	2	3	4	5
I= 1	0.4000	0.4000	0.4000	0.4000	0.4000
	*******				*******
I= 2	0.4000	0.4000	0.4012	0.4000	0.4000
I= 3	0.4000	0.4005	0.5032	0.4005	0.4000
			^^^^^^^		
I= 4	0.4000	0.4000	0.4012	0.4000	0.4000
I= 5	0.4000	0.4000	0.4000	0.4000	0.4000
	*******				*******

*** GAS SATURATION ****

	j= 1	2	3	4	5
I= 1	0.0531	0.0510	0.0509	0.0510	0.0531
	*******				*******
I= 2	0.0524	0.0511	0.0501	0.0511	0.0524
I= 3	0.0524	0.0518	0.0431	0.0518	0.0524
			^^^^^^^		
I= 4	0.0524	0.0511	0.0501	0.0511	0.0524
I= 5	0.0531	0.0510	0.0509	0.0510	0.0531
	*******				*******

**** WELL REPORT ****

ROW I	COLUMN J	OIL RATE (STB/D)	WATER RATE (STB/D)	GAS RATE (SCF/D)	GOR (SCF/STB)
1	1	-50.0	-1.1	-745.8	14.9156
5	1	-50.0	-1.1	-745.8	14.9156
1	5	-50.0	-1.1	-745.8	14.9156
5	5	-50.0	-1.1	-745.8	14.9156
3	3	0.0	200.0	0.0	0.0000

OILOLD = 0.299661D+06
WATOLD = 0.278306D+06
GASOLD = 0.152883D+09

OILNEW = 0.298661D+06
WATNEW = 0.279283D+06
GASNEW = 0.152414D+09

\+ FOR FLUID INJECTION
\- FOR FLUID PRODUCTION
OIL FLOW = -1000.0
WATER FLOW = 977.7
GAS FLOW =-0.470070D+06

OIL MATERIAL BALANCE =1.0000
WATER MATERIAL BALANCE =1.0000
GAS MATERIAL BALANCE =0.9995

NO. OF ADIP ITERATIONS = 13

NO. OF SATURATION LOOPS = 1
NO. OF PRESSURE LOOPS = 1
NO. OF MATERIAL BALANCE ITERATIONS = 1

PRESSURE AVERAGE =1895.2
OIL SATURATION AVERAGE =0.5444
WATER SATURATION AVERAGE =0.4043
GAS SATURATION AVERAGE =0.0514
TEMPERATURE AVERAGE =100.00
OIL RECOVERY IN THIS TIME STEP =0.3315

EXECUTION TIME (cpu) = 11.794270 SEC.

PRODUCTION DATA AFTER 4 TIME STEPS
TIME = 20.00 DAYS

*** OIL PRESSURE ****

	j= 1	2	3	4	5
I= 1	1888.4181	1894.1905	1897.6223	1894.1905	1888.418
	*******				*******
I= 2	1890.2630	1894.6613	1899.1530	1894.6613	1890.2630
I= 3	1890.8637	1894.9967	1905.3077	1894.9967	1890.8637
			^^^^^^^		
I= 4	1890.2630	1894.6613	1899.1530	1894.6613	1890.2630
I= 5	1888.4181	1894.1905	1897.6223	1894.1905	1888.418
	*******				*******

*** WATER PRESSURE ****

	j= 1	2	3	4	5
I= 1	1888.3485	1894.1209	1897.5527	1894.1209	1888.348
	*******				*******
I= 2	1890.1934	1894.5917	1899.0838	1894.5917	1890.1934
I= 3	1890.7941	1894.9273	1905.2553	1894.9273	1890.7941
			^^^^^^^		
I= 4	1890.1934	1894.5917	1899.0838	1894.5917	1890.1934
I= 5	1888.3485	1894.1209	1897.5527	1894.1209	1888.348
	*******				*******

*** GAS PRESSURE ****

	j= 1	2	3	4	5
I= 1	1888.4088	1894.1807	1897.6126	1894.1807	1888.408
	*******				*******
I= 2	1890.2535	1894.6515	1899.1437	1894.6515	1890.2535
I= 3	1890.8543	1894.9874	1905.3255	1894.9874	1890.8543
			^^^^^^^		
I= 4	1890.2535	1894.6515	1899.1437	1894.6515	1890.2535
I= 5	1888.4088	1894.1807	1897.6126	1894.1807	1888.408
	*******				*******

*** OIL SATURATION ****

	j= 1	2	3	4	5
I= 1	0.5465	0.5486	0.5484	0.5486	0.5465
	*******				*******
I= 2	0.5471	0.5485	0.5466	0.5485	0.5471
I= 3	0.5470	0.5464	0.4236	0.5464	0.5470
			^^^^^^^		
I= 4	0.5471	0.5485	0.5466	0.5485	0.5471
I= 5	0.5465	0.5486	0.5484	0.5486	0.5465
	*******				*******

*** WATER SATURATION ****

	j= 1	2	3	4	5
I= 1	0.4000	0.4000	0.4000	0.4000	0.4000
	*******				*******
I= 2	0.4000	0.4000	0.4026	0.4000	0.4000
I= 3	0.4000	0.4011	0.5348	0.4011	0.4000
			^^^^^^^		
I= 4	0.4000	0.4000	0.4026	0.4000	0.4000
I= 5	0.4000	0.4000	0.4000	0.4000	0.4000
	*******				*******

*** GAS SATURATION ****

	j= 1	2	3	4	5
I= 1	0.0536	0.0514	0.0516	0.0514	0.0536
	*******				*******
I= 2	0.0529	0.0515	0.0508	0.0515	0.0529
I= 3	0.0531	0.0525	0.0416	0.0525	0.0531
			^^^^^^^		
I= 4	0.0529	0.0515	0.0508	0.0515	0.0529
I= 5	0.0536	0.0514	0.0516	0.0514	0.0536
	*******				*******

**** WELL REPORT ****

ROW I	COLUMN J	OIL RATE (STB/D)	WATER RATE (STB/D)	GAS RATE (SCF/D)	GOR (SCF/STB)
1	1	-50.0	-1.1	-761.2	15.2232
5	1	-50.0	-1.1	-761.2	15.2232
1	5	-50.0	-1.1	-761.2	15.2232
5	5	-50.0	-1.1	-761.2	15.2232
3	3	0.0	200.0	0.0	0.0000

OILOLD = 0.298661D+06
WATOLD = 0.279283D+06
GASOLD = 0.152414D+09

OILNEW = 0.297661D+06
WATNEW = 0.280261D+06
GASNEW = 0.151944D+09

+ FOR FLUID INJECTION
- FOR FLUID PRODUCTION
OIL FLOW = -1000.0
WATER FLOW = 977.7
GAS FLOW =-0.470060D+06

OIL MATERIAL BALANCE =1.0000
WATER MATERIAL BALANCE =1.0000
GAS MATERIAL BALANCE =1.0000

NO. OF ADIP ITERATIONS = 13

NO. OF SATURATION LOOPS = 1
NO. OF PRESSURE LOOPS = 1
NO. OF MATERIAL BALANCE ITERATIONS = 1

PRESSURE AVERAGE =1893.6
OIL SATURATION AVERAGE =0.5425
WATER SATURATION AVERAGE =0.4057
GAS SATURATION AVERAGE =0.0518
TEMPERATURE AVERAGE =100.00
OIL RECOVERY IN THIS TIME STEP =0.3315

EXECUTION TIME (cpu) = 11.793963 SEC.

PRODUCTION DATA AFTER 5 TIME STEPS
TIME = 25.00 DAYS

*** OIL PRESSURE ****

	j= 1	2	3	4	5
I= 1	1886.7430	1892.5482	1896.0372	1892.5482	1886.743
	*******				*******
I= 2	1888.5907	1893.0117	1897.6068	1893.0117	1888.5907
I= 3	1889.1892	1893.3296	1904.5841	1893.3296	1889.1892
			^^^^^^^		
I= 4	1888.5907	1893.0117	1897.6068	1893.0117	1888.5907
I= 5	1886.7430	1892.5482	1896.0372	1892.5482	1886.743
	*******				*******

*** WATER PRESSURE ****

	j= 1	2	3	4	5
I= 1	1886.6733	1892.4786	1895.9676	1892.4786	1886.673
	*******				*******
I= 2	1888.5210	1892.9421	1897.5378	1892.9421	1888.5210
I= 3	1889.1196	1893.2603	1904.5353	1893.2603	1889.1196
			^^^^^^^		
I= 4	1888.5210	1892.9421	1897.5378	1892.9421	1888.5210
I= 5	1886.6733	1892.4786	1895.9676	1892.4786	1886.673
	*******				*******

*** GAS PRESSURE ****

	j= 1	2	3	4	5
I= 1	1886.7337	1892.5385	1896.0276	1892.5385	1886.733
	*******				*******
I= 2	1888.5813	1893.0020	1897.5981	1893.0020	1888.5813
I= 3	1889.1799	1893.3207	1904.6080	1893.3207	1889.1799
			^^^^^^^		
I= 4	1888.5813	1893.0020	1897.5981	1893.0020	1888.5813
I= 5	1886.7337	1892.5385	1896.0276	1892.5385	1886.733
	*******				*******

*** OIL SATURATION ****

	j= 1	2	3	4	5
I= 1	0.5460	0.5481	0.5477	0.5481	0.5460
	*******				*******
I= 2	0.5466	0.5480	0.5438	0.5480	0.5466
I= 3	0.546 4	0.5449	0.3956	0.5449	0.5464
			^^^^^^^		
I= 4	0.546 6	0.5480	0.5438	0.5480	0.5466
I= 5	0.5460	0.5481	0.5477	0.5481	0.5460
	*******				*******

*** WATER SATURATION ****

	j= 1	2	3	4	5
I= 1	0.4000	0.4000	0.4000	0.4000	0.4000
	*******				*******
I= 2	0.4000	0.4000	0.4048	0.4000	0.4000
I= 3	0.400 0	0.4020	0.5643	0.4020	0.4000
			^^^^^^^		
I= 4	0.400 0	0.4000	0.4048	0.4000	0.4000
I= 5	0.4000	0.4000	0.4000	0.4000	0.4000
	*******				*******

*** GAS SATURATION ****

	j= 1	2	3	4	5
I= 1	0.0540	0.0518	0.0523	0.0518	0.0540
	*******				*******
I= 2	0.0534	0.0520	0.0514	0.0520	0.0534
I= 3	0.0537	0.0532	0.0402	0.0532	0.0537
			^^^^^^^		
I= 4	0.0534	0.0520	0.0514	0.0520	0.0534
I= 5	0.0540	0.0518	0.0523	0.0518	0.0540
	*******				*******

**** WELL REPORT ****

ROW I	COLUMN J	OIL RATE (STB/D)	WATER RATE (STB/D)	GAS RATE (SCF/D)	GOR (SCF/STB)
1	1	-50.0	-1.1	-782.4	15.6478
5	1	-50.0	-1.1	-782.4	15.6478
1	5	-50.0	-1.1	-782.4	15.6478
5	5	-50.0	-1.1	-782.4	15.6478
3	3	0.0	200.0	0.0	0.0000

OILOLD = 0.297661D+06
WATOLD = 0.280261D+06
GASOLD = 0.151944D+09

OILNEW = 0.296661D+06
WATNEW = 0.281239D+06
GASNEW = 0.151473D+09

\+ FOR FLUID INJECTION
\- FOR FLUID PRODUCTION
OIL FLOW = -1000.0
WATER FLOW = 977.6
GAS FLOW =-0.470171D+06

OIL MATERIAL BALANCE =1.0000
WATER MATERIAL BALANCE =1.0000
GAS MATERIAL BALANCE =1.0001

NO. OF ADIP ITERATIONS = 13

NO. OF SATURATION LOOPS = 1
NO. OF PRESSURE LOOPS = 1
NO. OF MATERIAL BALANCE ITERATIONS = 1

PRESSURE AVERAGE =1892.0
OIL SATURATION AVERAGE =0.5406
WATER SATURATION AVERAGE =0.4071
GAS SATURATION AVERAGE =0.0523
TEMPERATURE AVERAGE =100.00
OIL RECOVERY IN THIS TIME STEP =0.3315

EXECUTION TIME (cpu) = 11.753837 SEC.

PRODUCTION DATA AFTER 6 TIME STEPS
TIME = 30.00 DAYS

*** OIL PRESSURE ****

	j= 1	2	3	4	5
I= 1	1885.0677	1890.9001	1894.4317	1890.9001	1885.067
	*******				*******
I= 2	1886.9184	1891.3575	1896.0457	1891.3575	1886.9184
I= 3	1887.5140	1891.6634	1903.7610	1891.6634	1887.5140
			^^^^^^^		
I= 4	1886.9184	1891.3575	1896.0457	1891.3575	1886.9184
I= 5	1885.0677	1890.9001	1894.4317	1890.9001	1885.067
	*******				*******

*** WATER PRESSURE ****

	j= 1	2	3	4	5
I= 1	1884.9981	1890.8305	1894.3621	1890.8305	1884.998
	*******				*******
I= 2	1886.8488	1891.2879	1895.9771	1891.2879	1886.8488
I= 3	1887.4444	1891.5942	1903.7157	1891.5942	1887.4444
			^^^^^^^		
I= 4	1886.8488	1891.2879	1895.9771	1891.2879	1886.8488
I= 5	1884.9981	1890.8305	1894.3621	1890.8305	1884.998
	*******				*******

*** GAS PRESSURE ****

	j= 1	2	3	4	5
I= 1	1885.0586	1890.8905	1894.4223	1890.8905	1885.058
	*******				*******
I= 2	1886.9091	1891.3479	1896.0378	1891.3479	1886.9091
I= 3	1887.5049	1891.6548	1903.7905	1891.6548	1887.5049
			^^^^^^^		
I= 4	1886.9091	1891.3479	1896.0378	1891.3479	1886.9091
I= 5	1885.0586	1890.8905	1894.4223	1890.8905	1885.058
	*******				*******

*** OIL SATURATION ****

	j= 1	2	3	4	5
I= 1	0.5455	0.5477	0.5470	0.5477	0.5455
	*******				*******
I= 2	0.5461	0.5475	0.5401	0.5475	0.5461
I= 3	0.5458	0.5430	0.3700	0.5430	0.5458
			^^^^^^^		
I= 4	0.5461	0.5475	0.5401	0.5475	0.5461
I= 5	0.5455	0.5477	0.5470	0.5477	0.5455
	*******				*******

*** WATER SATURATION ****

	j= 1	2	3	4	5
I= 1	0.4000	0.4000	0.4000	0.4000	0.4000
	*******				*******
I= 2	0.4000	0.4000	0.4079	0.4000	0.4000
I= 3	0.4000	0.4032	0.5912	0.4032	0.4000
			^^^^^^^		
I= 4	0.4000	0.4000	0.4079	0.4000	0.4000
I= 5	0.4000	0.4000	0.4000	0.4000	0.4000
	*******				*******

*** GAS SATURATION ****

	j= 1	2	3	4	5
I= 1	0.0545	0.0523	0.0530	0.0523	0.0545
	*******				*******
I= 2	0.0539	0.0525	0.0520	0.0525	0.0539
I= 3	0.0543	0.0538	0.0388	0.0538	0.0543
			^^^^^^^		
I= 4	0.0539	0.0525	0.0520	0.0525	0.0539
I= 5	0.0545	0.0523	0.0530	0.0523	0.0545
	*******				*******

**** WELL REPORT ****

ROW I	COLUMN J	OIL RATE (STB/D)	WATER RATE (STB/D)	GAS RATE (SCF/D)	GOR (SCF/STB)
1	1	-50.0	-1.1	-805.2	16.1038
5	1	-50.0	-1.1	-805.2	16.1038
1	5	-50.0	-1.1	-805.2	16.1038
5	5	-50.0	-1.1	-805.2	16.1038
3	3	0.0	200.0	0.0	0.0000

OILOLD = 0.296661D+06
WATOLD = 0.281239D+06
GASOLD = 0.151473D+09

OILNEW = 0.295661D+06
WATNEW = 0.282216D+06
GASNEW = 0.151003D+09

\+ FOR FLUID INJECTION
\- FOR FLUID PRODUCTION
OIL FLOW = -1000.0
WATER FLOW = 977.5
GAS FLOW =-0.470313D+06

OIL MATERIAL BALANCE =1.0000
WATER MATERIAL BALANCE =1.0000
GAS MATERIAL BALANCE =1.0001

NO. OF ADIP ITERATIONS = 13

NO. OF SATURATION LOOPS = 1
NO. OF PRESSURE LOOPS = 1
NO. OF MATERIAL BALANCE ITERATIONS = 1

PRESSURE AVERAGE =1890.4
OIL SATURATION AVERAGE =0.5388
WATER SATURATION AVERAGE =0.4085
GAS SATURATION AVERAGE =0.0527
TEMPERATURE AVERAGE =100.00
OIL RECOVERY IN THIS TIME STEP =0.3315

EXECUTION TIME (cpu) = 11.788455 SEC.

PRODUCTION DATA AFTER 7 TIME STEPS
TIME = 35.00 DAYS

*** OIL PRESSURE ****

	j= 1	2	3	4	5
I= 1	1883.3887	1889.2462	1892.8149	1889.2462	1883.388
	*******				*******
I= 2	1885.2427	1889.6990	1894.4784	1889.6990	1885.2427
I= 3	1885.8358	1889.9998	1902.7746	1889.9998	1885.8358
			^^^^^^^		
I= 4	1885.2427	1889.6990	1894.4784	1889.6990	1885.2427
I= 5	1883.3887	1889.2462	1892.8149	1889.2462	1883.388
	*******				*******

*** WATER PRESSURE ****

	j= 1	2	3	4	5
I= 1	1883.3191	1889.1766	1892.7453	1889.1766	1883.319
	*******				*******
I= 2	1885.1731	1889.6294	1894.4103	1889.6294	1885.1731
I= 3	1885.7662	1889.9308	1902.7323	1889.9308	1885.7662
			^^^^^^^		
I= 4	1885.1731	1889.6294	1894.4103	1889.6294	1885.1731
I= 5	1883.3191	1889.1766	1892.7453	1889.1766	1883.319
	*******				*******

*** GAS PRESSURE ****

	j= 1	2	3	4	5
I= 1	1883.3797	1889.2368	1892.8056	1889.2368	1883.379
	*******				*******
I= 2	1885.2335	1889.6896	1894.4715	1889.6896	1885.2335
I= 3	1885.8267	1889.9917	1902.8092	1889.9917	1885.8267
			^^^^^^^		
I= 4	1885.2335	1889.6896	1894.4715	1889.6896	1885.2335
I= 5	1883.3797	1889.2368	1892.8056	1889.2368	1883.379
	*******				*******

*** OIL SATURATION ****

j=	1	2	3	4	5
I= 1	0.5450	0.5472	0.5463	0.5472	0.5450
	*******				*******
I= 2	0.5456	0.5470	0.5355	0.5470	0.5456
I= 3	0.5452	0.5409	0.3470	0.5409	0.5452
			^^^^^^^		
I= 4	0.5456	0.5470	0.5355	0.5470	0.5456
I= 5	0.5450	0.5472	0.5463	0.5472	0.5450
	*******				*******

*** WATER SATURATION ****

j=	1	2	3	4	5
I= 1	0.4000	0.4000	0.4000	0.4000	0.4000
	*******				*******
I= 2	0.4000	0.4000	0.4120	0.4000	0.4000
I= 3	0.4000	0.4048	0.6154	0.4048	0.4000
			^^^^^^^		
I= 4	0.4000	0.4000	0.4120	0.4000	0.4000
I= 5	0.4000	0.4000	0.4000	0.4000	0.4000
	*******				*******

*** GAS SATURATION ****

j=	1	2	3	4	5
I= 1	0.0550	0.0528	0.0537	0.0528	0.0550
	*******				*******
I= 2	0.0544	0.0530	0.0525	0.0530	0.0544
I= 3	0.0548	0.0544	0.0376	0.0544	0.0548
			^^^^^^^		
I= 4	0.0544	0.0530	0.0525	0.0530	0.0544
I= 5	0.0550	0.0528	0.0537	0.0528	0.0550
	*******				*******

**** WELL REPORT ****

ROW I	COLUMN J	OIL RATE (STB/D)	WATER RATE (STB/D)	GAS RATE (SCF/D)	GOR (SCF/STB)
1	1	-50.0	-1.1	-828.9	16.5770
5	1	-50.0	-1.1	-828.9	16.5770
1	5	-50.0	-1.1	-828.9	16.5770
5	5	-50.0	-1.1	-828.9	16.5770
3	3	0.0	200.0	0.0	0.0000

OILOLD = 0.295661D+06
WATOLD = 0.282216D+06
GASOLD = 0.151003D+09

OILNEW = 0.294661D+06
WATNEW = 0.283194D+06
GASNEW = 0.150532D+09

\+ FOR FLUID INJECTION
\- FOR FLUID PRODUCTION
OIL FLOW = -1000.0
WATER FLOW = 977.5
GAS FLOW =-0.470473D+06

OIL MATERIAL BALANCE =1.0000
WATER MATERIAL BALANCE =1.0000
GAS MATERIAL BALANCE =1.0002

NO. OF ADIP ITERATIONS = 12

NO. OF SATURATION LOOPS = 1
NO. OF PRESSURE LOOPS = 1
NO. OF MATERIAL BALANCE ITERATIONS = 1

PRESSURE AVERAGE =1888.8
OIL SATURATION AVERAGE =0.5369
WATER SATURATION AVERAGE =0.4100
GAS SATURATION AVERAGE =0.0532
TEMPERATURE AVERAGE =100.00
OIL RECOVERY IN THIS TIME STEP =0.3315

EXECUTION TIME (cpu) = 11.881874 SEC.

PRODUCTION DATA AFTER 8 TIME STEPS
TIME = 40.00 DAYS

*** OIL PRESSURE ****

	j= 1	2	3	4	5
I= 1	1881.7044	1887.5867	1891.1869	1887.5867	1881.7044
	*******				*******
I= 2	1883.5618	1888.0357	1892.9068	1888.0357	1883.5618
I= 3	1884.1519	1888.3358	1901.6064	1888.3358	1884.1519
			^^^^^^^		
I= 4	1883.5618	1888.0357	1892.9068	1888.0357	1883.5618
I= 5	1881.7044	1887.5867	1891.1869	1887.5867	1881.7044
	*******				*******

*** WATER PRESSURE ****

	j= 1	2	3	4	5
I= 1	1881.6348	1887.5171	1891.1173	1887.5171	1881.6348
	*******				*******
I= 2	1883.4922	1887.9661	1892.8393	1887.9661	1883.4922
I= 3	1884.0823	1888.2671	1901.5669	1888.2671	1884.0823
			^^^^^^^		
I= 4	1883.4922	1887.9661	1892.8393	1887.9661	1883.4922
I= 5	1881.6348	1887.5171	1891.1173	1887.5171	1881.6348
	*******				*******

*** GAS PRESSURE ****

	j= 1	2	3	4	5
I= 1	1881.6955	1887.5773	1891.1778	1887.5773	1881.6955
	*******				*******
I= 2	1883.5528	1888.0264	1892.9010	1888.0264	1883.5528
I= 3	1884.1430	1888.3283	1901.6455	1888.3283	1884.1430
			^^^^^^^		
I= 4	1883.5528	1888.0264	1892.9010	1888.0264	1883.5528

I= 5	1881.6955	1887.5773	1891.1778	1887.5773	1881.6955
	*******				*******

*** OIL SATURATION ****

j=	1	2	3	4	5
I= 1	0.5445	0.5467	0.5456	0.5467	0.5445
	*******				*******
I= 2	0.5451	0.5465	0.5300	0.5465	0.5451
I= 3	0.5446	0.5384	0.3267	0.5384	0.5446
			^^^^^^^		
I= 4	0.5451	0.5465	0.5300	0.5465	0.5451
I= 5	0.5445	0.5467	0.5456	0.5467	0.5445
	*******				*******

*** WATER SATURATION ****

j=	1	2	3	4	5
I= 1	0.4000	0.4000	0.4000	0.4000	0.4000
	*******				*******
I= 2	0.4000	0.4001	0.4170	0.4001	0.4000
I= 3	0.4000	0.4067	0.6368	0.4067	0.4000
			^^^^^^^		
I= 4	0.4000	0.4001	0.4170	0.4001	0.4000
I= 5	0.4000	0.4000	0.4000	0.4000	0.4000
	*******				*******

*** GAS SATURATION ****

j=	1	2	3	4	5
I= 1	0.0555	0.0533	0.0543	0.0533	0.0555
	*******				*******
I= 2	0.0549	0.0535	0.0529	0.0535	0.0549
I= 3	0.0554	0.0549	0.0365	0.0549	0.0554
			^^^^^^^		
I= 4	0.0549	0.0535	0.0529	0.0535	0.0549
I= 5	0.0555	0.0533	0.0543	0.0533	0.0555
	*******				*******

**** WELL REPORT ****

ROW I	COLUMN J	OIL RATE (STB/D)	WATER RATE (STB/D)	GAS RATE (SCF/D)	GOR (SCF/STB)
1	1	-50.0	-1.1	-853.3	17.0658
5	1	-50.0	-1.1	-853.3	17.0658
1	5	-50.0	-1.1	-853.3	17.0658
5	5	-50.0	-1.1	-853.3	17.0658
3	3	0.0	200.0	0.0	0.0000

OILOLD = 0.294661D+06
WATOLD = 0.283194D+06
GASOLD = 0.150532D+09

OILNEW = 0.293661D+06
WATNEW = 0.284171D+06
GASNEW = 0.150062D+09

\+ FOR FLUID INJECTION
\- FOR FLUID PRODUCTION
OIL FLOW = -1000.0
WATER FLOW = 977.4
GAS FLOW =-0.470648D+06

OIL MATERIAL BALANCE =1.0000
WATER MATERIAL BALANCE =1.0000
GAS MATERIAL BALANCE =1.0002

NO. OF ADIP ITERATIONS = 11

NO. OF SATURATION LOOPS = 1
NO. OF PRESSURE LOOPS = 1
NO. OF MATERIAL BALANCE ITERATIONS = 1

PRESSURE AVERAGE =1887.1
OIL SATURATION AVERAGE =0.5350
WATER SATURATION AVERAGE =0.4114
GAS SATURATION AVERAGE =0.0536
TEMPERATURE AVERAGE =100.00
OIL RECOVERY IN THIS TIME STEP =0.3315

EXECUTION TIME (cpu) = 11.464670 SEC.